Realistic quantum fields with gauge and gravitational interaction emerge in the generic static structure

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We study a finite basic structure that possibly underlies the observed elementary quantum fields with gauge and gravitational interactions. Realistic wave functions of locally interacting quantum fields emerge naturally as low-resolution descriptions of the generic distribution of many quantifiable properties of arbitrary static objects. We prove that in any quantum theory with the superposition principle, evolution of a current state unavoidably continues along alternate routes with every Hamiltonian that possesses pointer states. Then for a typical system the Hamiltonian changes unpredictably during evolution. This applies to the emergent quantum fields too. Yet the Hamiltonian is unambiguous for isolated emergent systems with sufficient symmetry, e.g., local supersymmetry. The other emergent systems, without specific physical laws, cannot be inhabitable. The acceptable systems are eternally inflating universes with reheated regions. We see how eternal inflation perpetually creates new short-scale physical degrees of freedom and why they are initially in the ground state. In the emergent quantum worlds probabilities follow from the first principles. The Born rule is not universal but there are reasons to expect it in a typical world. The emergent quantum evolution is necessarily Everettian (many-world). However, for a finite underlying structure the Everett branches with the norm below a positive threshold cease to exist. Hence some experiments that could be motivated by taking the Everett view too literally will be fatal for the participants.

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I. INTRODUCTION

Quantum mechanics enjoys the status of one of today's most impeccable and yet the most counterintuitive phys-

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ical theory. It has flawlessly passed about a century of accurate experimental tests. So deeply has it permeated the contemporary physical picture that in the search for the ultimate "theory of everything" many physicists regard quantum mechanics as a basic postulate. Popular approaches with this assumption include string theory and loop quantum gravity.

At the same time, ever since its inception, quantum mechanics has seemed to defy logic. Even now, not every issue of its internal consistency is resolved to the satisfaction of the entire physics community.

Major progress in "making sense" of quantum mechanics occurred as early as 1950s with the remarkable thesis by Hugh Everett [1]. Everett demonstrated that quantum mechanics does not necessarily require the awkward postulate of probabilistic (moreover, by Bell's theorem [2], nonlocal and superluminal) collapse of a wave function during measurement. Altogether, Everett pointed out a straightforward albeit still bewildering to many people fact that with the universal applicability of unitary quantum dynamics to both microscopic and macroscopic systems the physical world unavoidably splits into numerous co-existing branches of "alternate realities."

Despite this progress and subsequent developments, notably understanding of quantum decoherence [3–7], open questions persist about quantum mechanics as well as about other fundamental principles of nature. The questions that have been raised include the following:

1. Are Everett's alternate worlds real, or does the wave function nevertheless collapse to a single macroscopic configuration because of yet unknown nonlocal [2] physics [8–12]?

2. If all the Everett branches exist, why do we find ourselves in a given branch with probability that is proportional to the squared norm of the branch's wave function [13–18]?

3. Can we understand the remaining quantum-mechanical postulates and the physical Hamiltonian in deeper, more elementary and natural terms [19]?

4. Why do we observe the specific laws of physics [20, 21]?

5. Why do these laws, in particular, the parameters of the Standard Model, as far as observations show [22], remain constant in time and space?

6. Why does time in quantum mechanics conceptually differ from space, yet time and space unify in the relativistic description of nature?

7. How are the new microscopic degrees of freedom created when the universe expands eternally in cosmological inflation [23, 24]? Can inflation ultimately "run out" of the new small-scale degrees of freedom?

8. Why do the infinitely numerous short-scale degrees of freedom appear during inflation almost in the ground state, necessary for inflation to proceed [25–28]?

9. How to reconcile the unitarity of quantum evolution with the apparent loss of information during evaporation of black holes by the nearly thermal Hawking radiation [29, 30]? How to explain the "firewall" paradox [31]?

We answer these and several other fundamental questions by observing intriguing logical consequences of well-tested physical laws in the established range of their validity. This paper shows that elementary structures, natural or mathematical, that are typically encountered in our experience, generically contain emerging phenomenologically viable quantum-field worlds. These worlds experience inflation and subsequent evolution that resembles and possibly includes ours. In this Introductory section we visualize the eventual simple results with a suggestive sketch. The reader should nevertheless remember that the following is only a helpful but not precise illustration of results that we obtain systematically in the main sections.

Let us consider harmonic modes of a weakly coupled field during cosmological inflation. The modes whose frequency substantially exceeds the rate of the Hubble expansion (the Hubble constant) should be almost in the ground state for the inflation to be possible [cf. eq. (199)]. The wave function of the ground state for the modes of the field with negligible interaction is close to the Gaussian form. Specifically, the wave function of M modes $\{m\}$ is

$$\psi(q) \approx \prod_m \left(\pi^{-\frac{1}{4}}\, e^{-\frac{1}{2}\, q_m^2}\right) = \pi^{-\frac{M}{4}}\, e^{-\frac{1}{2}\sum_m q_m^2} \tag{1}$$

where q_m are the appropriately normalized amplitudes of the field modes. In Sec. VII we will see that the quantum fields that emerge in the described generic structure have gauge and gravitational interactions. Their ground state may not at all be close to the Gaussian form (1). Yet let us continue this illustration under the assumption of the initial wave function (1).

Is there a simple, familiar structure that produces an objectively existing function of the form (1)? In fact, almost any collection of many objects does, regardless of what the word "object" stands for. It is sufficient that the objects or their groups possess quantifiable properties q which, when measured in appropriate coordinates, are distributed by eq. (1). This is expected generically by the central limit theorem of probability theory. Of course, properties q^p of many physical or mathematical objects familiar to us obey a non-Gaussian distribution that is specific to the objects' nature. However, general linear combinations of many independent properties, $q^n = \sum_p c_p^n\, q^p$, are indeed distributed by the universal normal (Gaussian) law, at least, under the conditions of

the central limit theorem. Such N generically chosen uncorrelated coordinates q^n are therefore distributed by eq. (1). In this paper we consider the generic collection of objects and a finite number N of their generic properties q^n that satisfy the central limit theorem. We call this system a "generic structure".

For a large but finite collection of the basic objects, the precise distribution $\nu(q)$ of their properties q is a sum of delta-functions:

$$\nu(q) = \sum_a \delta^{(N)}(q - q_a)\,. \tag{2}$$

Here $q_a \equiv \{q_a^n\}$ is a vector of the N considered properties q^n that are evaluated at the a'th object. When we view this distribution at a finite resolution Δq such that the range Δq contains many basic objects, we observe a smoothed distribution that approaches the universal Gaussian form. As discussed in Sec. III and illustrated in Fig. 1, we can generally describe the smoothed distribution by linear combinations of linearly-independent smooth basis functions. The linear space spanned by the coefficients of these linear combinations will be found to give rise to the Hilbert space of quantum states of fields with local dynamical symmetries. The complex structure of the Hilbert space and unique Hermitian product, related to *objective* physical probability in the emergent world, appear automatically. Quantum entanglement over arbitrary distances in this structure is trivial (Sec. X) because the physical degrees of freedom then fundamentally are coordinates of the basic distribution. An entangled quantum state of two dynamical variables matches to a distribution term that is localized in the both corresponding coordinates. Locality of the field dynamics in the emergent spacetime also follows as shown in this paper.

The available continuous transformations of the basis functions, superposed to smoothly fit the basic distribution, may seem to vastly outnumber the transformations that constitute the quantum evolution with a specific Hamiltonian. Yet the identical concern can also be raised for the standard axiomatic quantum mechanics. As Sec. II shows, the quantum superposition principle entails that unitary transformation of the wave function by *any* Hamiltonian that possesses evolving pointer states [4, 7] creates physically existing branches of evolution. In particular, if the superposition principle holds at least on the experimentally tested scales then any quantum states (or, in the Heisenberg picture, the operators for the observables) should apparently undergo physically real evolution by numerous Hermitian operators in the role of the system Hamiltonian. Then the Hamiltonian for a typical observer would unpredictably change during evolution. It thus becomes especially puzzling why the experiments and our daily experience indicate the evolution by a rather special, due to its symmetries, Hamiltonian that is constant throughout time and space.

A key to answering why our perceived world evolves by spacetime-independent and special physical laws may be the following. *Local* dynamical symmetries, including the gauge and diffeomorphism symmetries, not only are symmetries of the action (governing evolution of the wave function) but also are symmetries of the wave function itself. This has long been recognized for quantum gravity [32] but seldom discussed for quantum field theory, where for calculations it is convenient to fix the gauge and also to work not with the wave function but with field correlation functions.

Let us again consider the described earlier smoothed distribution of quantifiable properties of objects. We regard this distribution as the basic entity that materially represents the wave function of an emergent physical system. The emergent physical local fields $\hat{\phi}^\alpha(\boldsymbol{x})$ are certain linear combinations of uncorrelated coordinates $\hat{q}^n$ of the basic distribution (Sec. IV). We identify gauge fields $\boldsymbol{A}(\boldsymbol{x})$ with parameters that distinguish possible directions of the local evolution of $\hat{\phi}^\alpha(\boldsymbol{x})$ (Sec. V). A wave function $\psi[\phi^\alpha(\boldsymbol{x}), \boldsymbol{A}(\boldsymbol{x})]$ of a gauge-symmetric system is necessarily constant among gauge-equivalent configurations (Appendix A). Therefore, let us identify the coordinates $q = \{q^n\}$ with the cosets of gauge-equivalent configurations $[\phi^\alpha(\boldsymbol{x}), \boldsymbol{A}(\boldsymbol{x})]$ (Sec. V). Then the smoothed distribution of the generic properties materially represents the wave function ψ of an emergent system with the gauge symmetry. The coordinates q become the dynamical variables of this emergent physical system.

The specified emergent quantum fields can evolve only by a gauge-invariant action. These fields are not merely a theoretical invention. Their wave function exists objectively as a tangible distinctive structure among the objects that surround us and among many other abstract or material objects. In the same generic collections of basic objects we could attempt to identify other emergent "wave functions" without the local symmetry. Yet they would not represent antropically viable physical worlds with definite laws of internal physical evolution.[1] Without concrete dynamical laws that do not change in time unpredictably, the internal systems in the respective worlds cannot evolve biologically and develop naturally into intelligent observers. Fortunately, those unacceptable emergent "quantum systems" have a configuration space of different dimensionality. Therefore, they do not blend with an emergent system that possesses a sufficiently[2] restrictive local symmetry and definite physical

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[1] More generally, several authors noted [20, 21] that any mathematical structure possibly corresponds to some emergent physical system. In relation to this general statement, the present paper explores which mathematical structures and their "material" representations actually *can* be behind our physical world, and which of the acceptable structures are the most likely (generic) ones. We will also see that the mathematical structure of the physical laws is insufficient to predict every phenomenological outcome, which sometimes depends on the material representation of this mathematics.

[2] We will see that gauge and diffeomorphism symmetries are insufficient for fixing the physical laws. Yet this task is achieved

laws. The emergent symmetric quantum field system is thus a distinctive, objectively existing entity.

Gravitational interaction arises in the emergent systems similarly to the gauge interaction. The role of the gauge symmetry is then fulfilled by the diffeomorphism symmetry. Gravity, however, involves important technical subtleties. We treat them in Secs. VI and VII.

In essence, the described description of the generic Gaussian distribution develops into a complicated wave function of an inflating universe.[3] Despite the simple Gaussian form of the original distribution, the resulting quantum world can be rather convoluted. In many aspects this corresponds to the well understood transformation of the simple, nearly Gaussian wave function of the inflationary field modes into the contemporary cosmic structure.

The world that we perceive today is only one of the numerous decoherent branches into which the global wave function splits during its inflationary and later evolution. If fundamentally the global wave function is a smoothed representation of a *finite* underlying structure then the wave function has intrinsic uncertainty because the same discrete basic distribution allows slightly different but equally statistically significant smooth fits. (As an analogy, there is uncertainty in the smooth boundary of any familiar macroscopic object, which below certain resolution is composed of almost point-like elementary particles.) Once a particular Everett's branch becomes unresolvable at this intrinsic uncertainty of the overall wave function, the branch can no longer be regarded as an objectively existing part of the actual fundamental structure. Hence the norm of every physical Everett's branch should exceed a positive limit, fixed for a given basic structure (Sec. III). This has very important real-world consequences, not only for remote future but for our present (Sec. IX).

In addition, since the norm of every physical Everett's branch exceeds a fixed positive threshold, the number of the alternate branches becomes finite. Then the probability of various macroscopic outcomes of a quantum process is unambiguous. We therefore do not need to impose the Born rule as a postulate, which the Born rule is in any current interpretation of quantum mechanics, including Everett's (e.g. [18]). Rather we can explore under which conditions the Born rule arises naturally for the emergent quantum systems (Sec. IX).

The rest of the paper is organized as follows. The end of Introduction specifies our notations. Sec. II considers any quantum theory (either axiomatic or of a deeper origin) for which the quantum superposition principle holds at least on the scales probed by our experiments. The section demonstrates that then any current quantum state typically starts branches in which the current observables subsequently evolve by many various time-dependent Hamiltonians. We answer why we see our world evolving by an unchanged Hamiltonian in later sections.

Sec. III describes how an evolving wave function with connection to probability emerges naturally as a continuous set of fitting functions for the distribution of properties in the generic static collection of almost arbitrary objects.

Secs. IV-VIII show in detail that some emergent systems of Sec. III objectively exist. Namely, they are clearly and objectively identifiable: there are no infinitesimally modified emergent systems with which the considered emergent systems could blend. These emergent systems correspond to specific states of quantum fields with gauge-invariant and diffeomorphism-invariant evolution and inflationary initial conditions.

Specifically, Sec. IV describes the natural emergence of a complex wave function, of probability-related Hermitian product, and of linear evolution of the wave function with a Hamiltonian operator. Sec. V considers gauge symmetries and introduces emergent gauge fields. Its results will be crucial for answering why, despite the arguments of Sec. II, we experience evolution by a special and unchanged Hamiltonian (Sec. VIII). Sec. VI studies gravitational degrees of freedom. A reader who is not interested in technicalities may prefer to skip Secs. IV-VI. Then Sec. VII describes emergence of field states representing a universe that expands from inflationary past. At last, Sec. VIII discusses mechanisms that restrict the physical evolution to a specific Hamiltonian whose couplings do not vary in spacetime and with the experiments that measure them. While these properties are implicit for the "physical laws" based on our experience, they should, as highlighted by Sec. II, be extremely surprising for quantum dynamics regardless of its possible more fundamental origin.

Sec. IX studies the probabilities for different macroscopic outcomes of a quantum process. When the basic structure that underlies the emergent physical systems is finite then these probabilities are objective and, in principle, unambiguous. The section arrives at important and, to the author's knowledge, previously unrecognized consequences of practical relevance.

Concluding Sec. X briefly summarizes the results. The paper also has three technical Appendixes. They will be referred to when their material is needed in the main sections.

A companion paper [33] describes quantitatively the full evolution and Hawking evaporation of black holes, including their central singularity and the final moment of the evaporation, by applying the generic first-principle construction of interacting quantum fields and physical spacetime that is developed in the present paper.

by local supersymmetry, Sec. VIII.

[3] We will find in Sec. VII E that the continuous evolution of the wave function by the physical Hamiltonian may be preceded by an initial transformation of the Gaussian function into a non-Gaussian ground state wave function of the field modes for this Hamiltonian.

A. Notations

We employ the units with $\hbar = c = m_P = 1$, where the Planck mass m_P is related to the Newton gravitational constant G as

$$m_P^2 \equiv \frac{1}{8\pi G}\,. \tag{3}$$

We use Greek indices $\mu, \nu, \chi, \dots$ for the components of spacetime tensors, and Latin indices $i, j, k, \dots$ for the components of spatial tensors. We label the components of field multiplets with Greek indices $\alpha, \beta, \gamma, \dots$ but we apply Latin indices $a, b, c, \dots$ to distinguish the adjoint-representation multiplets of gauge fields. We denote the components of general vectors or configuration-space coordinates with indices n and m.

We take the signature for the spacetime metric to be $(-,+,+,+)$ and parameterize the metric by the Arnowitt-Deser-Misner (ADM) decomposition [34, 35]

$$ds^2 = -(N dt)^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt). \tag{4}$$

Then the metric tensor and its inverse are

$$g_{\mu\nu} \;=\; \begin{pmatrix} -N^2 + N_i N^i & N_i \\ N_i & \gamma_{ij} \end{pmatrix}, \tag{5}$$

$$g^{\mu\nu} \;=\; \frac{1}{N^2}\begin{pmatrix} -1 & N^i \\ N^i & N^2\gamma^{ij} - N^i N^j \end{pmatrix}. \tag{6}$$

Indices of spatial tensors are lowered and raised by the spatial metric γ_{ij} and its inverse γ^{ij}. We denote differentiation by a comma, covariant differentiation based on the spacetime metric $g_{\mu\nu}$ by a semicolon, and covariant differentiation based on the spatial metric γ_{ij} by a vertical bar "|".

For the Fourier transformation of a function $\psi(q)$, $q \in \mathbb{R}^N$, we introduce a shorthand notation

$$đ^N p \equiv \frac{d^N p}{(2\pi)^N}\,. \tag{7}$$

Then

$$\psi(q) \;=\; \int đ^N p\; e^{ip\cdot q}\,\psi(p)\,, \tag{8}$$

$$\psi(p) \;=\; \int d^N q\, e^{-ip\cdot q}\,\psi(q)\,. \tag{9}$$

Also,

$$\int đ^N p\; e^{ip\cdot(q-q')} = \delta^{(N)}(q - q'), \tag{10}$$

where $\delta^{(N)}(q)$ is the N-dimensional Dirac delta function. Likewise we define

$$\bar{\delta}^{(N)}(p) \equiv (2\pi)^N \delta^{(N)}(p), \tag{11}$$

for which

$$\int đ^N p'\, f(p')\; \bar{\delta}^{(N)}(p' - p) = f(p) \tag{12}$$

and

$$\int d^N q\, e^{i(p'-p)\cdot q} = \bar{\delta}^{(N)}(p' - p). \tag{13}$$

Throughout the paper q and p denote "dynamical variables," describing the physical degrees of freedom in the configuration and momentum representations respectively. The dynamical variables q should be distinguished from spacetime coordinates x or spatial coordinates $\boldsymbol{x}$ of field operators, e.g., in $\hat{\phi}(x) \equiv \hat{\phi}(t, \boldsymbol{x})$. Likewise, the canonical dynamical momenta p should not be confused with the spatial wavevector $\boldsymbol{k}$ for the Fourier modes of field operators, e.g., in

$$\hat{\phi}(\boldsymbol{x}) = \int đ^3 k\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\, \hat{\phi}(\boldsymbol{k}). \tag{14}$$

We will use function notations for functions of a finite number of variables as well as for functionals (functions of functions). The quantum-mechanical wave function will be denoted by $\psi(f)$ and called "wave function" even when its argument f ranges over a set of field configurations $\{f(\boldsymbol{x})\}$. Since we may approximate a continuous function $f(\boldsymbol{x})$ by its discretization $\{f_n\}$ on a series of progressively refined grids, a functional $F[f(\boldsymbol{x})]$ can indeed be regarded as the limit of regular functions of an increasingly large number of variables. This view of functionals will underpin our fundamental construction of Sec. III, erasing physical distinction between functions and functionals.

Accordingly, $\int df$ will denote both a conventional and functional integral with a measure df. The Dirac delta function $\delta(f)$ is defined for any—discrete, continuous, or function—argument f by

$$\int df'\, \delta(f - f')\, F(f') \equiv F(f) \tag{15}$$

for any map $F(f)$.

II. FREEDOM OF QUANTUM EVOLUTION

Experiments and observations provide solid evidence that quantum principles apply to the physical world from at least the smallest scales probed by the particle colliders to macroscopic and even cosmological distances. The validity of quantum description on the accessible microscopic scales is verified, for example, by the precision measurements of renormalization running of the coupling constants and other parameters of the Standard Model due to quantum radiative corrections. At larger scales numerous experiments confirm accurate and detailed predictions of atomic and condensed matter physics, relying

on quantum description of electrons and the electromagnetic field. On macroscopic scales quantum mechanics is also tested with precision in, e.g., SQUID systems and quantum optics. Even on the largest observable cosmic scales quantum entanglement and the superposition principle are strongly suggested by the success of the inflationary paradigm, explaining the observed angular power spectra of cosmological inhomogeneities by inflationary amplification of the vacuum quantum fluctuations.

In this section we highlight a peculiar but logically unavoidable consequence of the cornerstone superposition principle of quantum theory at the accessible energies. This observation will guide us toward identifying in Secs. III-VIII realistic dynamical quantum field systems as emergent phenomena in a generic set of almost arbitrary static objects.

Let us consider quantum degrees of freedom that are described by commuting field operators $\hat{f}^\alpha(\boldsymbol{x})$. Here the discrete label α denotes a field type or/and the component of a field multiplet (e.g., a spin or isospin projection). The continuous label $\boldsymbol{x}$ of the field operators belongs to a 3-dimensional manifold. It is physically interpreted as the spatial coordinate in some coordinate frame. In this paper we discuss bosonic (commuting) fields. Fermionic (anticommuting) fields also naturally exist within the fundamental construction that is established in the sections below. We delegate a detailed description of the fermions and their evolution, with natural possibility for local supersymmetry, to a later paper.

Since the field operators $\hat{f}^\alpha(\boldsymbol{x})$ commute, we can expand any pure quantum state $|\psi\rangle$ over their simultaneous eigenstates:

$$|\psi\rangle = \int df\, \psi(f)\, |f\rangle\,, \tag{16}$$

with $\psi(f) \equiv \psi[f^\alpha(\boldsymbol{x})]$, $df \equiv \prod_\alpha [df^\alpha(\boldsymbol{x})]$, and $\hat{f}^\alpha(\boldsymbol{x})|f\rangle = f^\alpha(\boldsymbol{x})|f\rangle$. Normalizing the mutually orthogonal eigenstates $|f\rangle$ to the delta function of eq. (15),

$$\langle f'|f\rangle = \delta(f'-f)\,, \tag{17}$$

we arrive at representing the state $|\psi\rangle$ by a wave function $\psi(f)$. By eqs. (16–17), the scalar product of quantum states in the Hilbert space of wave functions is

$$\langle\psi_1|\psi_2\rangle = \int df\, \psi_1^*(f)\, \psi_2(f)\,. \tag{18}$$

We may define the operators of canonical momenta fields $\hat{\pi}_\alpha(\boldsymbol{x})$ that are conjugate to the fields $\hat{f}^\alpha(\boldsymbol{x})$ by

$$\hat{\pi}_\alpha(\boldsymbol{x})\, \psi(f) \equiv -i\, \frac{\delta}{\delta f^\alpha(\boldsymbol{x})}\, \psi(f)\,. \tag{19}$$

The operators $\hat{f}^\alpha(\boldsymbol{x})$ and $\hat{\pi}_\alpha(\boldsymbol{x})$ obey the canonical commutation relations

$$[\hat{f}^\alpha(\boldsymbol{x}), \hat{\pi}_\beta(\boldsymbol{y})] = i\delta^\alpha_\beta \delta^{(3)}(\boldsymbol{x}-\boldsymbol{y})\,, \tag{20}$$

where δ^α_β and $\delta^{(3)}(\boldsymbol{x})$ are respectively the Kronecker symbol and Dirac delta function.

We now consider an arbitrary analytic function H' of the operators $\{\hat{f}^\alpha(\boldsymbol{x})\}$ and $\{\hat{\pi}_\alpha(\boldsymbol{x})\}$:

$$\begin{aligned} H'(\hat{f},\hat{\pi}) = \sum_n \int & d^3x_1\, d^3x_2 \ldots d^3y_1\, d^3y_2 \ldots \times \\ & \times\ K_{n\,\alpha_1\alpha_2\ldots}^{\beta_1\beta_2\ldots}(\boldsymbol{x}_1,\boldsymbol{x}_2,\ldots;\boldsymbol{y}_1,\boldsymbol{y}_2,\ldots) \times \\ & \times\ \hat{f}^{\alpha_1}(\boldsymbol{x}_1)\hat{f}^{\alpha_2}(\boldsymbol{x}_2)\ldots\hat{\pi}_{\beta_1}(\boldsymbol{y}_1)\hat{\pi}_{\beta_2}(\boldsymbol{y}_2)\ldots\,. \end{aligned} \tag{21}$$

To avoid ambiguities, we take only the functions H' such that the sum (21) has a finite number of terms and these terms contain a finite number of operators $\hat{f}^\alpha$ and $\hat{\pi}_\beta$. Also we use only the kernels K_n that yield well-defined convergent integrals in $H'(\hat{f},\hat{\pi})\,\psi$. For an infinitesimal parameter dt we define

$$\hat{U} \equiv \exp(-i\hat{H}'dt) = \hat{1} - i\hat{H}'dt + O(dt^2)\,, \tag{22}$$

where $\hat{H}' \equiv H'(\hat{f},\hat{\pi})$ of eq. (21) and $\hat{1}$ is the identity operator. Further, let

$$\begin{aligned} \hat{f}'^\alpha(dt,\boldsymbol{x}) &\equiv \hat{U}^{-1}\hat{f}^\alpha(\boldsymbol{x})\,\hat{U}\,, \\ \hat{\pi}'_\alpha(dt,\boldsymbol{x}) &\equiv \hat{U}^{-1}\hat{\pi}_\alpha(\boldsymbol{x})\,\hat{U}\,. \end{aligned} \tag{23}$$

The similarity transformation (23) is canonical, i.e., it preserves the commutation relations (20).

Now let us require $\hat{H}'$ to be Hermitian: $\hat{H}'^\dagger = \hat{H}'$. The hermiticity imposes some straightforward constraints on the kernels K_n in eq. (21) but the remaining freedom for the choice of these kernels is still vast. The corresponding operator $\hat{U}$ then becomes unitary: $\hat{U}^{-1} = \hat{U}^\dagger$. Hence for any function $O(\hat{f},\hat{\pi})$ and any state $|\psi\rangle$

$$\begin{aligned} &\langle\psi|\,O(\hat{f}',\hat{\pi}')\,|\psi\rangle = \langle\psi|\,\hat{U}^{-1}\,O(\hat{f},\hat{\pi})\,\hat{U}\,|\psi\rangle = \\ &= \langle\psi'|\,O(\hat{f},\hat{\pi})\,|\psi'\rangle \end{aligned} \tag{24}$$

with

$$|\psi'\rangle = \hat{U}\,|\psi\rangle\,. \tag{25}$$

If $\hat{H}'$ coincides with the physical Hamiltonian $\hat{H}$ of a real-world system of fields then in the Schrodinger picture of quantum mechanics the system that is initially in a state $|\psi\rangle$ after a time span dt evolves into the state (25).

A. Freedom of evolution in the Heisenberg picture

In the Heisenberg picture of quantum mechanics $|\psi\rangle$ is static. Physical evolution is then manifested by the change of the operators for observables, e.g., the energy-momentum tensor, density of currents, field-strength tensors, or their averages over finite regions. An observable that at a time t corresponds to an operator

$$\hat{O} = O(\hat{f},\hat{\pi}) \tag{26}$$

at a new time $t + dt$ is matched to

$$\hat{O}' = \hat{U}^{-1}\, O(\hat{f},\hat{\pi})\, \hat{U} = O(\hat{f}',\hat{\pi}')\,, \tag{27}$$

where $\hat{f}'$ and $\hat{\pi}'$ are given by eqs. (23). The physical state at the new time is the same state $|\psi\rangle$ that now is regarded as a linear combination of the eigenstates of the evolved observable $\hat{O}'$.

We now make our first *key observation*. Let a Hermitian operator $\hat{H}'$ of eq. (21) *differ* from the physical Hamiltonian $\hat{H}$. Even then the operator $\hat{O}'$ of eq. (27), obtained with $\hat{H}'$ through eq. (22), is a valid Hermitian operator that is connected to the original observable $\hat{O}$ by a continuous group of similarity transformations, parameterized by $dt \in \mathbb{R}$. For several observables $\hat{O}_\alpha$ the corresponding $\hat{O}'_\alpha$, obtained through these similarity transformations, satisfy the same commutation relations as the original observables do. Then $\hat{O}'_\alpha$ are operators for the original observables that have evolved by our arbitrary Hermitian operator $\hat{H}'$, used as the Hamiltonian.

One may insist that in axiomatic quantum mechanics a wave function should not be considered independently from a specific Hamiltonian of the studied physical system. Yet even then we can in principle arrange a measurement of $|\psi\rangle$ projections on the eigenstates of $\hat{O}'_\alpha$, provided that all the fields in these operators are localized to the region of observation. Let $|1'\rangle$ and $|2'\rangle$ be such eigenstates with different eigenvalues. During the suggested measurement the wave function of a studied subsystem (e.g., $|\psi\rangle = |1'\rangle + |2'\rangle$) and the rest of the system ($|\text{ext}\rangle$, representing the measuring device, observer and environment) changes as

$$|\psi\rangle|\text{ext}\rangle = \Big(|1'\rangle + |2'\rangle\Big)|\text{ext}\rangle \to |1'\rangle|\text{ext}_{1'}\rangle + |2'\rangle|\text{ext}_{2'}\rangle\,.$$

After a typical measurement the macroscopic states $|\text{ext}_{1'}\rangle$ and $|\text{ext}_{2'}\rangle$ decohere. Therefore, all subsequent observations in the considered system become confined to either only the first or only the second branch on the right-hand side of the above equation. These observations thus become identical to those expected for the evolution of the original system $|\psi\rangle$ by the Hamiltonian $\hat{H}'$. Importantly, this applies even when the system is by definition described by a different Hamiltonian $\hat{H}$.

Most of the new "Hamiltonians" $\hat{H}'$ do not possess pointer states [4, 7], stable to decoherence and representing valid Everett's branches. Yet for a typical system we could consider many $\hat{H}'$ that have pointer states but deviate from $\hat{H}$. For example, we could apply Hamiltonians where particle masses and couplings change slowly from their current values. If the quantum superposition principle is a law of nature then the projections of $|\psi\rangle$ on the evolving pointer states for the alternative Hamiltonian $\hat{H}'$ represent the Everett branches of the evolution that this $\hat{H}'$ generates. These branches should be as real for their observers that will evolve from our current state as our branch of evolution will be for our future evolution.

Why, despite the presented arguments, the experiments consistently indicate quantum evolution by only a single and very specific Hamiltonian $\hat{H}$ that is constant in time and space? We emphasize that we cannot resolve this paradox by assuming a multitude of worlds that are governed by all the conceivable Hamiltonians [20, 21]. The question concerns our own world. Why of all the choices for quantum evolution that *materialize here and now*, we live through only highly specific quantum dynamics that remains unchanged throughout spacetime?

B. Repeat in the Schrodinger picture

It is instructive and necessary for our further progress to reformulate the same question in the framework of the Schrodinger picture. Let at a time t the system be in a pure state with a wave function $\psi(f)$. By the Born rule of standard quantum mechanics, $|\psi(f)|^2$ is the probability density in the configuration space $\{f\}$. We now consider a function $\psi'(f')$ that is obtained by some invertible convolution of $\psi(f)$ with a kernel $E(f',f)$:

$$\psi'(f') = \int df\, E(f',f)\,\psi(f). \tag{28}$$

For any wave function $\psi(f)$

$$\int df\, |\psi(f)|^2 = \int df'\, |\psi'(f')|^2 \tag{29}$$

whenever

$$\int df'\, E^*(f',f_1)\, E(f',f_2) = \delta(f_1 - f_2). \tag{30}$$

The last condition states that the convolution with the kernel $E^\dagger(f',f) \equiv E^*(f,f')$ is the inverse of the convolution (28).

Property (29) amounts to unitarity of the transformation (28) with respect to the Hermitian product (18). An infinitesimal unitary transformation (25) with $\hat{U} = \exp[-iH'(\hat{f},\hat{\pi})\,dt]$ can also be presented in the convolution form (28). This is achieved by the Feynman path integral, yielding for an infinitesimal dt

$$\psi'(f') = \int df\, đ\pi\, e^{i[(f'-f)\cdot\pi - H'(f,\pi)\,dt]}\,\psi(f)\,. \tag{31}$$

Here

$$f\!\cdot\!\pi \equiv \int d^3x \sum_\alpha f^\alpha(\boldsymbol{x})\,\pi_\alpha(\boldsymbol{x}) \tag{32}$$

and the measure $đ\pi$ for a finite number of dynamical variables is defined by eq. (7). When the wave function depends on a function argument $f = \{f^\alpha(\boldsymbol{x})\}$ then the canonical momentum measure $đ\pi$ is defined by the condition

$$\int đ\pi\, e^{i(f'-f)\cdot\pi} = \delta(f'-f)\,, \tag{33}$$

generalizing eq. (10). We can verify the equivalence of the Schrodinger transformation (25) and the path integral (31) by noting that the evolving wave functions for both cases satisfy the same first-order differential equation

$$\frac{\partial}{\partial t}\,\psi' = -i\hat{H}'\,\psi' \tag{34}$$

with the same initial condition $\psi'|_{dt=0} = \psi$.[4]

We now reformulate our first "key observation" in the Schrodinger picture of quantum evolution. Let ψ be the wave function of a physical system at a time t. Then we consider its equivalent representation ψ' from eq. (28) using *any* convolution with the property (30). Such a convolution can be obtained from an arbitrary Hamiltonian function $H'(f,\pi)$ by the path integral (31). Let $\hat{O} = O(\hat{f},\hat{\pi})$ be a Hermitian operator for an observable. At the time t the probability of various values of this observable equals the squared norm of the projections of ψ on the $\hat{O}$ eigenstates. But we are free to associate ψ' and the squares of its projections on the $\hat{O}$ eigenstates with a different time moment t'. Given the existence of ψ, its transform (28) also exists (as the presentation of the same physical state in another basis of the quantum Hilbert space). This transform, i.e. another possible presentation of the same state, is the Schrodinger-picture wave function that has evolved from t to t' by the arbitrarily chosen Hamiltonian H'.

III. WAVE FUNCTION AS A FIT FOR THE GENERIC UNDERLYING STRUCTURE

We now show that extremely simple, generic in nature and ubiquitous in mathematics, static basic systems contain as objectively existing composite entities the wave functions of emergent physical systems. These wave functions evolve unitarily and are related to probability of outcomes of quantum processes observed by internal inhabitants of the emergent systems. By studying instead of an abstract, axiomatically defined wave function the concrete tangible quantity with the identical properties but simple real-world implementations, we will be able to track down answers to several long-standing fundamental questions, including those listed in Sec. I. We will also resolve why, despite the results of Sec. II, we observe as the physical evolution only relatively few linear transformations that are generated by the fixed physical Hamiltonian.

Consider a large but finite set of objects of any nature and enumerate these objects by natural numbers $a = 1, 2, \dots, A$. Suppose that every object of the set can be characterized by a large number of properties. Consider a finite number N of the properties and enumerate them by $n = 1, 2, \dots, N$. Also suppose that these properties can be quantified by real numbers q_a^n:

$$q_a^n \equiv \begin{pmatrix}\text{value of } n\text{-th property}\\ \text{for } a\text{-th object}\end{pmatrix}. \tag{35}$$

As one of countless examples for the respective q_a^n, we may think of all the quantifiable characteristics n of all the planets a in the visible universe. We should, however, remember that while this or other large collections of macroscopic physical objects do contain emergent evolving systems whose internal physical laws coincide with those of our physical world, the latter is unlikely to emerge from its own objects. Nor we expect the basic fundamental objects belong to any similar world with space and time. Instead, it is more natural for the basic structure that gives rise to our physical world to exist as a self-contained static entity, unrelated to any external spacetime or physical dynamics. We will discuss this in Conclusion, Sec. X.

Let $q \equiv (q^1, q^2, \dots) \equiv \{q^n\}$ be a vector of particular values of the properties. Let $\nu(q)$ be the density of the distribution of these values for the considered set of the objects a:

$$\nu(q) = \sum_a \delta(q - q_a)\,. \tag{36}$$

In other words, let the number of objects whose properties fall into a range Δq be

$$\Delta a = \int_{\Delta q} dq\ \nu(q)\,, \tag{37}$$

where $dq \equiv \prod_n dq^n$. For a chosen resolution Δq, suppose that an interval of width Δq at any point q contains many objects a. I.e., let in any such interval $\Delta a \gg 1$ (see Fig. 1.a). In addition, suppose that the relative change in Δa for adjacent intervals is insignificant. Then at the resolution Δq we can approximate the exact distribution density $\nu(q)$ (36) by a smooth real fitting function $\rho(q)$. This means that for any window function $W(q)$ with characteristic width no smaller than Δq and for any point q_0, the smooth approximation $\rho(q)$ satisfies

$$\int dq\, W(q - q_0)\,\nu(q) \simeq \int dq\, W(q - q_0)\,\rho(q)\,. \tag{38}$$

Fig. 1.a shows that we may view $\rho(q)$ as the distribution $\nu(q)$ that is binned over cells Δq. The cells are small in comparison to the width of $W(q)$ yet every cell still contains many objects. On the other hand, as illustrated by Fig. 1.b, we can also construct the smoothed equivalent of the basic distribution by fitting it with a superposition of smooth fitting functions $f_Q(q,t)$. Here t labels various choices of a basis $\{f_Q(q)\}$ for the fitting

[4] $\hat{H}' \equiv H'(\hat{f},\hat{\pi})$ may contain products of non-commuting operators $\hat{f}$ and $\hat{\pi}$. Then for the equivalence of eqs. (25) and (31) we order these products in the Hamiltonian operator (22) so that all the momenta $\hat{\pi}$ stand to the right, as shown in eq. (21).

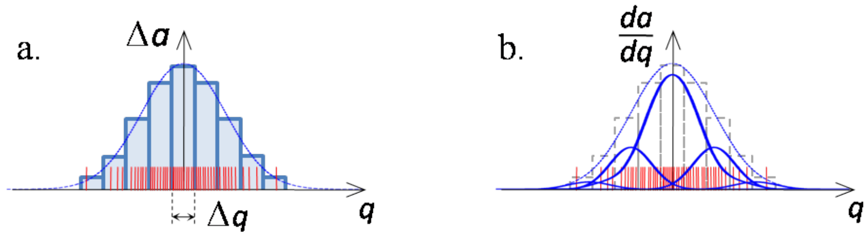

FIG. 1: An example of a one-dimensional distribution $\nu(q)$ of a certain property that can be quantified by a real value q (red vertical lines). Panel a. shows the properties' values q for various objects of some collection. These properties are binned over intervals of equal width Δq. The resulting histogram is fitted by a smooth function. Panel b. shows that the same distribution can be described above certain resolution by a linear superposition of smooth functions of a specified shape with varying numeric coefficients, spanning a real linear space. Its natural transition to the complex Hilbert space of quantum mechanics is described in Sec. IV. Secs. V-VII then demonstrate emergence of a specific Hamiltonian that for a similar multidimensional $\nu(q)$ (Fig. 2 on p. 13) generates an isolated group of isomorphism transformations of the respective Hilbert space. They constitute valid physical evolution, which can be that of our observed world.

functions, whereas Q distinguishes the function members of any particular basis. Thus we match

$$\nu(q) \to \rho(q) \equiv \sum_Q \rho(Q,t)\, f_Q(q,t) \tag{39}$$

so that eq. (38) holds. Importantly, $f_Q(q,t)$ are not required to be localized around any particular value of q.

It may be tempting to associate the linear space spanned by the linear combinations of the functions $f_Q(q)$ from eq. (39) for a given t with the linear space of quantum-mechanical wave functions. A change in the basis for the fitting functions, $\{f_Q\}_t \to \{f'_Q\}_{t'}$, would then constitute transition to a different representation of the prospective wave function $\rho(Q,t) \to \rho(Q,t')$. However, besides a minor and soon to be naturally lifted issue of ρ being real while the physical wave function being complex, the norm of a wave function should yield the physical probability.

The norm is specified by a Hermitian product $\langle 1|2\rangle$ for the functions that represent any physical states 1 and 2. One might think of $\int dq\, \rho_1(q)\, \rho_2(q)$ as a "natural" candidate for the product of ρ_1 and ρ_2; however, this assignment is unmotivated and, moreover, unacceptable. The quantity $\int dq\, \rho_1\rho_2$ is different for various subjective choices of the arbitrary coordinates q. Therefore, it cannot specify the objective[5] physical probability, unrelated to our description of the system. Since the Hermitian product of quantum mechanics has unequivocal physical meaning—it determines the probability of observing a particular branch of the wave function after a measurement—to understand the emergence of the physical Hermitian product, let us understand the emergence of the probability.

Suppose that in the course of physical evolution along a continuous trajectory of alternative complete sets of the basis functions $f_Q(q,t)$ in decomposition (39) a fitting function $\rho(Q,t)$ splits in, for simplicity, two terms $\rho_1 + \rho_2$. Each of the terms represents a possible future branch that looses coherence with the other one. By repeating the splitting process R times, we arrive at a superposition of 2^R outcomes:

$$\rho = \rho_{11\ldots1} + \rho_{21\ldots1} + \cdots + \rho_{22\ldots2}\,. \tag{40}$$

Let the splits be caused by quantum measurement, e.g., determining a projection of election spin. Correspondingly, an individual term $\rho_{r_1 r_2 \ldots r_R}$ (with $r_i \in \{1,2\}$) in eq. (40) describes the measured system and its environment, including the observer, in the branch with a specific sequence of measured results, $(r_1, r_2, \ldots, r_R)$, Ref. [1]. Then the alternate branches, represented by the different terms on the right-hand side of eq. (40), should be mutually orthogonal in the Hilbert space of the corresponding emerging quantum mechanics. Indeed, first, a typical quantum measurement splits the wave function of the measured system along different non-degenerate eigenstates of a Hermitian operator; therefore, these eigenstates are necessarily orthogonal. For example, a measurement of the vertical projection of the election spin yields the orthogonal spin-up and spin-down states. Second, for a system with many "environmental" degrees of freedom, decoherence ensures that the products of different Everett's branches contribute to the density matrix of the system negligibly [7].

Let $\rho(q)$ be the linear combination (39) of smooth functions $f_Q(q,t)$ that fits best the fundamental distribution $\nu(q)$. Assuming a large but finite number of the basic objects $\{a\}$, let us bin the fundamental distribution $\nu(q)$, as illustrated by Fig. 1.a. Without limiting the generality, we take bins of equal width Δq. We suppose that the number of the objects Δa_b in a bin b fluctuates

[5] Refs. [13–16] suggested that the probability may, on the contrary, be subjective. We will nevertheless see below that there exists objective, description-independent probability for various macroscopic branches of quantum evolution in the emergent systems.

over the bins about a smoothly changing value

$$\overline{\Delta a}_b = \bar\rho(q_b)\,\Delta q \tag{41}$$

with the variance $\sigma^2(\Delta a_b)$. The fluctuations can be caused by deterministic defining properties of the structure $\nu(q)$, by the assignment of the numeric values of q, by random shot noise, or by any of these reasons combined. For the finite set of the basic objects there is an intrinsic ambiguity in the value of the fitting coefficients $\rho(Q,t)$ and of the fitting function $\rho(q)$. At most, we can find the "best fitting function" that minimizes some artificially chosen statistics. A simple, convenient choice is the χ^2 statistics

$$\chi^2 = \sum_b \frac{[\Delta a_b - \rho(q_b)\,\Delta q]^2}{\sigma^2(\Delta a_b)}\,. \tag{42}$$

With a different choice of bins, the best fit will somewhat differ. However, let us consider only the binning choices for which $\Delta a_b \gg 1$ yet Δq_b are smaller than the characteristic variation scales of $\rho(q)$. Then either a fit $\rho(q)$ is rejected at a statistically significant level for every such binning, or it is an acceptable fit for every of them. Thus within some unavoidable intrinsic uncertainty the notion of $\rho(q_b)$ fitting an underlying structure $\nu(q)$ is objective.

Let us introduce the variance density

$$v(q) \equiv \frac{\sigma^2(\Delta a)}{\Delta q}\,. \tag{43}$$

Then the χ^2 statistics (42) equals

$$\chi^2 = \sum_b \frac{\Delta q}{v(q_b)}\left[\frac{\Delta a_b}{\Delta q} - \rho(q_b)\right]^2. \tag{44}$$

When Δa for adjacent bins are uncorrelated then $v(q)$ of eq. (43) is independent of the bin width Δq. Then Δa_b is determined, at least locally, by the Poisson process with the expectation value $\overline{\Delta a}_b$ of eq. (41). We assume this from now on.

Consider a function

$$\rho(q) = \bar\rho(q) + \delta\rho(q)\,, \tag{45}$$

where $\bar\rho(q)$ and $\delta\rho(q)$ vary insignificantly over an interval Δq, and

$$\bar\rho(q_b) = \overline{\Delta a}_b/\Delta q\,. \tag{46}$$

Since by the last equation $\langle \Delta a_b/\Delta q - \bar\rho(q_b)\rangle = 0$, eq. (44) gives

$$\chi^2(\rho) = \chi^2(\bar\rho) + \sum_b \frac{\Delta q}{v(q_b)}\,[\delta\rho(q_b)]^2\,. \tag{47}$$

We replace the sum in the last term of eq. (47) by an integral, yielding

$$\delta\chi^2(\rho) \equiv \chi^2(\rho) - \chi^2(\bar\rho) \simeq \int \frac{dq}{v(q)}\,[\delta\rho(q)]^2\,. \tag{48}$$

Let evolution in t split an overall fitting function ρ in two terms $\rho = \rho_1 + \rho_2$ that eventually decohere. If the terms ρ_1 and ρ_2 are orthogonal in the parameterization-independent[6] scalar product

$$\langle 1|2\rangle = \int \frac{dq}{v(q)}\,\rho_1(q)\,\rho_2(q) \tag{49}$$

then $\langle\rho|\rho\rangle = \langle 1|1\rangle + \langle 2|2\rangle$. More generally, for repeated measurements and multiple mutually orthogonal outcomes ρ_i

$$\langle\rho|\rho\rangle = \sum_{\text{outcomes } i} \langle i|i\rangle\,. \tag{50}$$

Importantly, when a new branch ρ_i "thins" to the extent that its substitution to eq. (48) as $\delta\rho$ results in a change $\delta\chi^2$ below a statistical significance limit $\delta\chi^2_{\rm min}$ then the respective state $|i\rangle$ does not exist objectively. Here the threshold value $\delta\chi^2_{\rm min}$ is defined as a borderline between two qualitatively different situations:

a. Removal of ρ_i from the sum over alternate decoherent branches, e.g. in eq. (40), changes the confidence level of the overall fit ρ by an order of unity,

vs.

b. Removal of the branch ρ_i does not affect the confidence level of the fit ρ substantially.

It may be helpful to reformulate this as follows. The discrete basic distribution $\nu(q)$ cannot be described by a smooth approximation $\rho(q)$ beyond certain accuracy because once some threshold accuracy is exceeded, the fitting function can no longer be smooth since the underlying discrete distribution $\nu(q)$ is not.

What happens with a new evolution branch ρ_i when

$$\langle i|i\rangle \lesssim \delta\chi^2_{\rm min}\,, \tag{51}$$

where $\delta\chi^2_{\rm min}$ is the intrinsic uncertainty of unambiguous determination of the fitting function ρ? After decoherence of the alternate macroscopic outcomes, such

[6] The right-hand side of eq. (49) is invariant under a change of the configuration-space coordinates $q = \{q^n\}$, quantifying the properties $\{n\}$. Indeed, after a coordinate change $q \to q'(q)$, we have

$$dq' = J\,dq\,,$$

where $J \equiv |\partial q'/\partial q|$ is the Jacobian of the coordinate transformation. Then, since $da = \rho\,dq = \rho'\,dq'$,

$$\rho' = J^{-1}\rho\,.$$

The variance density $v(q)$ transforms as

$$v' = \frac{d\sigma^2}{dq'} = J^{-1} v\,.$$

Thus the right-hand side of eq. (49) is manifestly invariant under the coordinate change.

a branch does not present an objectively existing path of the system evolution. A quantum state ρ_i may be discussed mathematically but it has no objective representation through the objects of the basic structure. We thus established that the squared norm of every physically meaningful term in eq. (50) should exceeds a positive threshold $\delta\chi^2_{\rm min}$.

Any candidate for the physical probability should possess the conservation property (50). In the considered emergent quantum systems, however, we *calculate* the frequentist probability of the macroscopic physical outcomes instead of postulating it. This will be the subject of further Sec. IX.

We may build a full and consistent with experiment quantum field theory of the observed world using the scalar product (49). (Provided that we confirm its relation to the physical probability.) Historically, however, the scalar product of quantum mechanics has been chosen in the "canonical" form

$$\langle 1|2\rangle = \int dq\, \psi_1(q)\, \psi_2(q)\,. \tag{52}$$

(So far we use real functions ψ because the fitting function (39) is real. The next section shows that the standard complex wave function and canonical Hermitian product arise for the emergent quantum systems automatically.)

We convert the formulation of a quantum theory with the parameterization-independent scalar product (49) to the standard formulation with the canonical product (52) by introducing a *wave function*

$$\psi_i(q) \equiv v^{-1/2}(q)\, \rho_i(q)\,. \tag{53}$$

By definition, $v(q)$ is the variance density (43) for the overall distribution $\nu(q)$. Since the factor $v^{-1/2}$ is common to all the branches i, the functions ψ_i evolve linearly if and only if ρ_i do. The fit (39) in terms of ψ reads

$$\nu(q) \to \rho(q) = \sum_Q \psi(Q,t)\, F_Q(q,t)\,, \tag{54}$$

where

$$F_Q(q) = f_Q(q)\, v^{1/2}(q)\,. \tag{55}$$

For arbitrary branches 1 and 2 at any given t the product (49) becomes

$$\langle 1|2\rangle = \sum_{Q,Q'} \psi_1(Q)\, M_{QQ'}\, \psi_2(Q') \tag{56}$$

where

$$\begin{aligned} M_{QQ'} &= \int \frac{dq}{v(q)}\, F_Q(q)\, F_{Q'}(q) = \\ &= \int dq\, f_Q(q)\, f_{Q'}(q)\,. \end{aligned} \tag{57}$$

The canonical form (52) follows whenever

$$M_{QQ'} = \int dq\, f_Q\, f_{Q'} = \delta_{QQ'}\,, \tag{58}$$

i.e., whenever $f_Q(q)$ is the convolution kernel of an orthogonal transformation.

We may view the wave function $\psi(q)$ of eq. (53) as a special case of the fitting coefficients $\rho(Q)$ in the linear combination (39). Indeed, eq. (53) follows when in eq. (54) we set $F_Q(q) = \delta(Q-q)\, v^{1/2}(q)$. Then by eq. (55) we have $f_Q(q) = \delta(Q-q)$. This is one of infinitely many basis choices that bring $M_{QQ'}$ in the scalar product (56) to the canonical identity-matrix form.

IV. EVOLVING WAVE FUNCTION AND FIELD OPERATORS

We continue to explore emergent quantum systems whose wave function by construction is a low-resolution representation of a generic distribution. In this section we manifestly see that these emergent systems automatically possess such standard quantum-field-theoretical ingredients as a complex wave function, dynamical field operators (acting linearly on the wave function), and probability-related Hermitian product of the emergent quantum states of the field operators. We also consider a simple example of dynamical evolution for such a system. This example cannot describe a reasonable physical world but based on it we will find phenomenologically suitable emergent systems in subsequent Secs. V-VIII.

A. Emergence of a complex wave function

The generic basic structure from Sec. III gives rise to a quantum field theory as follows. Consider the emergent wave function $\psi(q)$ in any representation where the scalar product has the canonical form (52). There exists its dual representation that is formed by the superposition of the "waves on $\psi(q)$," the waves that furnish the irreducible representations of an abelian group of coordinate shifts

$$q \to q' = q - \Delta q, \quad \Delta q = \text{const}\,. \tag{59}$$

The wave function $\psi(q)$ in the dual representation is described by a two-component function of $p = (p_1, p_2, \ldots, p_N)$

$$\psi(p) \equiv \begin{pmatrix} \psi^r(p) \\ \psi^i(p) \end{pmatrix} \tag{60}$$

as

$$\begin{aligned} \psi(q) &= \int đp\, (\cos q{\cdot}p\,,\, -\sin q{\cdot}p) \begin{pmatrix} \psi^r(p) \\ \psi^i(p) \end{pmatrix} = \\ &= \int đp\, \mathrm{Re}\left[e^{iq\cdot p}\, \psi(p)\right]. \end{aligned} \tag{61}$$

Above,

$$i \equiv \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad q{\cdot}p \equiv \sum_n q^n p_n \,, \tag{62}$$

and the real-part function Re selects the upper component of its two-component argument. Under a shift of coordinates q (59) the values of $\psi(p)$ transform as

$$\psi(p) \to e^{i\Delta q\cdot p}\, \psi(p)\,. \tag{63}$$

Therefore, the two components ψ^r and ψ^i of $\psi(p)$ for every p form a 2-d real (1-d complex) representation of the abelian group (59).

We may as well consider a representation that is dual to the dual representation (60). In other words, we expand $\psi(p)$ over the "waves on $\psi(p)$" that transform irreducibly under the shifts of the coordinates p

$$p \to p' = p - \Delta p, \quad \Delta p = \text{const}\,. \tag{64}$$

This expansion,

$$\psi(p) = \int dq\, e^{-iq\cdot p}\, \psi(q)\,, \tag{65}$$

leads naturally to the complex extension of the original real wave function $\psi(q)$. This is a manifestation of the Pontryagin duality for the irreducible representations of an abelian group.

In the physics language, for a real $\psi(q)$, eq. (61) defines not the entire $\psi^r(p)$ and $\psi^i(p)$ but only their parts that are respectively even and odd under $p \to -p$ reflection. Any odd contribution to $\psi^r(p)$ does not change the left-hand side of eq. (61), and neither does any even contribution to $\psi^i(p)$. Our emergent physical states are the decoherent terms of the wave function natural splits by its physical evolution as in eq. (40):

$$\psi = \psi_{11\ldots1} + \psi_{21\ldots1} + \cdots + \psi_{22\ldots2}\,. \tag{66}$$

We continue to refer to two-outcome splits; the generalization to more than two orthogonal outcomes is straightforward. In the momentum representation the overall sum for $\psi(p) \equiv \psi^r(p) + i\psi^i(p)$ in eq. (66), for a real $\psi(q)$, satisfies the constraint $\psi(p) = \psi^*(-p)$. However, this constraint for a global complex $\psi(p)$ does not restrict the individual terms on the right-hand side of eq. (66), corresponding to individual Everett's branches, which are isolated in their future evolution dynamically. Then we also need to represent the unconstrained complex $\psi(p)$ of the individual terms in the q-space. Their q-space representation is provided by a two-component or, equivalently, a complex function

$$\psi(q) = \begin{pmatrix} \psi^r(q) \\ \psi^i(q) \end{pmatrix} = \psi^r(q) + i\psi^i(q)\,. \tag{67}$$

This complex function is connected to $\psi(p)$ by eq. (65) and equals

$$\psi(q) = \int đp\; e^{iq\cdot p}\, \psi(p) \in \mathbb{C}\,. \tag{68}$$

We thus showed that by treating the q- and p-space representations of the fitting functions (54) on equal footing and by accounting for the Everett branching process (66) we necessarily arrive at representing the individual decohered branches of quantum evolution by a complex wave function (67).

Yet not every linear transformation of the real wave function ψ due to changing the basis in eq. (54) is linear on the complex linear space. Let $\psi \to \psi'$ with

$$\psi' = \hat{A}\psi\,, \tag{69}$$

standing for

$$\begin{pmatrix} \psi'^r \\ \psi'^i \end{pmatrix} = \begin{pmatrix} \hat{A}^r_r & \hat{A}^r_i \\ \hat{A}^i_r & \hat{A}^i_i \end{pmatrix} \begin{pmatrix} \psi^r \\ \psi^i \end{pmatrix} \tag{70}$$

where $\psi^{(\prime)r,i}$ are real and the operators $\hat{A}^\alpha_\beta$ are linear. Complex linearity,

$$\hat{A}\,(c_1\psi_1 + c_2\psi_2) = c_1\hat{A}\psi_1 + c_2\hat{A}\psi_2 \tag{71}$$

for all $c_1,\, c_2 \in \mathbb{C}$, holds if and only if the operator $\hat{A}$ commutes with the matrix i (62):

$$\hat{A}\, i = i\, \hat{A}\,. \tag{72}$$

Thus for the complex linearity of operators it is necessary and sufficient that

$$A^r_r = A^i_i\,, \quad \hat{A}^r_i = -A^i_r\,. \tag{73}$$

All the configuration-space coordinate and momentum operators

$$\hat{q}^n = q^n \quad \text{and} \quad \hat{p}_n = -i\partial/\partial q^n \tag{74}$$

satisfy this condition. Therefore, they are linear on the complex linear space of the wave functions (67). The same applies to any multinomial or analytic function of the operators $\hat{q}^n$ and $\hat{p}_n$.

B. Hermitian product

We now identify the unique probability-related Hermitian product on the complex linear space of the dynamically isolated terms of eq. (66) (or of a similar equation for more than two decoherent outcomes of a quantum process). Consider $\langle\psi|\psi\rangle$ in a representation where the scalar product (49), which quantifies the state capacity for future splits before the state stops representing anything objectively existing, has the canonical form (52). For a real wave function $\psi(q)$ of eq. (61)

$$\begin{aligned} \langle\psi|\psi\rangle &= \int dq\, \psi^2(q) = \int đp\, \left\{[\psi^r(p)]^2 + [\psi^i(p)]^2\right\} = \\ &= \int đp\, |\psi(p)|^2\,. \end{aligned} \tag{75}$$

Likewise for a complex wave function $\psi(q)$ of eq. (68),

$$\langle\psi|\psi\rangle = \int đp\, |\psi(p)|^2 = \int dq\, |\psi(q)|^2 . \tag{76}$$

This measure of an emergent physical state objectively quantifies the goodness of fit (54). As discussed in Sec. IX, it determines the frequentist probability for an intrinsic observer in the emergent system to follow the particular Everett branch.

We look for a scalar product of complex wave functions (67) in the general form

$$\langle\psi_1|\psi_2\rangle = (\psi_1^r, \psi_1^i) \begin{pmatrix} \hat{M}_{rr} & \hat{M}_{ri} \\ \hat{M}_{ir} & \hat{M}_{ii} \end{pmatrix} \begin{pmatrix} \psi_2^r \\ \psi_2^i \end{pmatrix} \in \mathbb{C}\,, \tag{77}$$

and we require two following properties:

1. The product (77) reduces to the physical measure (76) for $\psi_1 = \psi_2$.

2. This product is linear in its second argument on the *complex* linear space (67):

$$\langle\psi_1|c\,\psi_2\rangle = c\,\langle\psi_1|\psi_2\rangle \qquad \forall c \in \mathbb{C}\,. \tag{78}$$

These two requirements uniquely determine the linear operators $\hat{M}_{\alpha\beta}$ in eq. (77), yielding the canonical Hermitian product

$$\langle\psi_1|\psi_2\rangle = \int dq\, \psi_1^*(q)\, \psi_2(q)\,, \tag{79}$$

where the star denotes complex conjugation.

C. "Generic" wave function

Even without specifying the origin(s) of the underlying basic structure, we anticipate that the fitting function $\rho(q)$ of the distribution of its properties q takes a certain generic form. We, however, warn the reader that the corresponding "generic" wave function ψ_0 from eq. (82) below *is not necessarily* the initial wave function of the short-scale modes that appear from the Planck scale during inflation. Their initial wave function is instead determined by the considerations presented in Sec. VII E. Whether or not we may call ψ_0 "the global wave function of the universe" is a matter of terminology. The actual picture, studied in Sec. VII E, is more subtle.

Let $q = \{q^n\}$ be values of many quantifiable properties for a huge collection of objects of any nature. Then the central limit theorem suggests that we may expect the generic distribution of the properties to be Gaussian. Of course, many (if not most) objects familiar to us are distributed not by the Gaussian law. However, general linear combinations of values of uncorrelated properties of the familiar objects are Gaussian, at least, within the conditions described by the central limit theorem of probability theory. In other words, for randomly selected $\nu(q)$

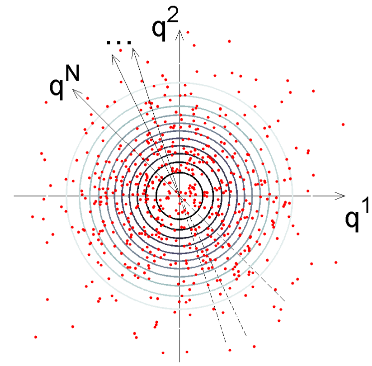

FIG. 2: Visualization of a generic distribution of N arbitrary properties, and its fitting by a generic Gaussian function. The coordinates $q^3, \ldots, q^N$, extending along other dimensions, are orthogonal mutually and orthogonal to the depicted plane (q^1, q^2).

that describe some underlying basic structure, the fitting function $\rho(q)$ has the generic form

$$\rho_0 = A^2 e^{-q^2}. \tag{80}$$

Here $q^2 \equiv \sum_n (q^n)^2$ where q^n are appropriately normalized uncorrelated linear combinations of the original coordinates (35), Fig. 2, and a positive parameter A follows from the normalization of the generic distribution.

Since for the locally-Poisson process, considered in Sec. III,

$$v(q) \equiv \frac{d\sigma^2}{dq} = \rho(q)\,, \tag{81}$$

the corresponding wave function (53) for the entire structure is also Gaussian:

$$\psi_0 = A\, e^{-q^2/2}. \tag{82}$$

This "generic" wave function, obtained from the fitting function of the generic collection of arbitrary objects a, has two suggestive properties. First, it *would* describe the initial ground state (the Bunch-Davies vacuum [36]) of the field modes during inflation if the fields did not interact. Second, it is invariant under the standard Fourier transformation (9). Moreover, it is also invariant under the fractional Fourier transformation (83-84), forming a continuous group that arises naturally in the considered structure as seen next.

D. Evolving field operators

Let us start from a single degree of freedom $q \in \mathbb{R}$ with a wave function $\psi(q)$. Consider a Lie group of transformations that continuously connects $\psi(q)$ with $\psi(p)$:

$$\psi \to \psi' = e^{-i\hat{H}d\varphi}\psi \tag{83}$$

where

$$\hat H = \frac{1}{2}\left(\hat p^2 + \hat q^2 - 1\right), \tag{84}$$

$$\hat q\,\psi(q) = q\,\psi(q), \quad \hat p\,\psi(q) = -i\frac{\partial}{\partial q}\,\psi(q)\,. \tag{85}$$

The infinitesimal Schrodinger transformation (83) is equivalent to the following Heisenberg transformation of the operators $\hat q$ and $\hat p$:

$$\begin{aligned} \hat q \;&\rightarrow\; e^{i\hat H d\varphi}\,\hat q\,e^{-i\hat H d\varphi} = \hat q + i[\hat H, \hat q]\,d\varphi = \hat q + \hat p\,d\varphi\,,\\ \hat p \;&\rightarrow\; e^{i\hat H d\varphi}\,\hat p\,e^{-i\hat H d\varphi} = \hat p + i[\hat H, \hat p]\,d\varphi = \hat p - \hat q\,d\varphi\,. \end{aligned} \tag{86}$$

Then for a finite value of the transformation parameter φ

$$\begin{aligned} \hat q \;&\rightarrow\; \hat q'_\varphi = \hat q\cos\varphi + \hat p\,\sin\varphi\,,\\ \hat p \;&\rightarrow\; \hat p'_\varphi = -\hat p\,\sin\varphi + \hat q\cos\varphi\,. \end{aligned} \tag{87}$$

This Lie group of transformations includes the Fourier transformation ($\varphi = \pi/2$) and the inverse Fourier transformation ($\varphi = -\pi/2$). Since the Hamiltonian (84) annihilates the Gaussian wave function ψ_0 (82), the infinitesimal transformation (83) and the entire Lie group generated by it leave ψ_0 invariant.

Let us now investigate if the basic structure from Sec. III could represent a free field theory. The emergent generic wave function (82) might be considered for giving rise to the vacuum state of a free field in a finite region of space through the following simple construction. For simplicity, we set the space $\{\boldsymbol{x}\}$ to be a three-dimensional cube $0 \le x, y, z \le L$ with coordinate volume $V = L^3$. We impose the periodic boundary conditions on the cube sides, i.e., try to construct a quantum field in a spatial region with torus topology.

Consider the following Hermitian linear combinations of the basic operators $\hat q^n$ and $\hat p_n$ of eq. (74):

$$\hat\phi(\boldsymbol{x}) \equiv \frac{1}{V^{1/2}}\sum_{\boldsymbol{m}} \frac{1}{\sqrt{2\omega_{\boldsymbol{m}}}}\,(\hat q^c_{\boldsymbol{m}}\cos \boldsymbol{k_m}\!\cdot\!\boldsymbol{x}\; + \\ + \;\hat q^s_{\boldsymbol{m}}\sin \boldsymbol{k_m}\!\cdot\!\boldsymbol{x}) \tag{88}$$

where

$$\boldsymbol{k_m} \equiv \frac{2\pi}{L}\,\boldsymbol{m}\,, \tag{89}$$

$\boldsymbol{m} = (m_1, m_2, m_3)$ runs over the sets of three integers with $|m_{1,2,3}| < M/2$, and $\hat q^c_{\boldsymbol{m}}$ and $\hat q^s_{\boldsymbol{m}}$ are the operators of various uncorrelated coordinates q^n of the system from Sec. III, illustrated by Fig. 2.[7] We will specify the positive real numbers $\omega_{\boldsymbol{m}}$ in eq. (88) later. In this and the next section, until we promote the spacetime metric to a dynamical degree of freedom in Sec. VI, for any spatial vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ by definition $\boldsymbol{a}\!\cdot\!\boldsymbol{b} \equiv \sum_i a_i b_i$.

By $\boldsymbol{k} \to -\boldsymbol{k}$ symmetry of the cosine and antisymmetry of the sign terms in eq. (88) we can symmetrize $\hat q^c_{\boldsymbol{m}}$ and antisymmetrize $\hat q^s_{\boldsymbol{m}}$ in $\boldsymbol{m} \to -\boldsymbol{m}$ without changing their sum. Therefore, without limiting generality, let us require that

$$\hat q^c_{-\boldsymbol{m}} = \hat q^c_{\boldsymbol{m}}\,, \quad \hat q^s_{-\boldsymbol{m}} = -\hat q^s_{\boldsymbol{m}}\,. \tag{90}$$

The operators $\hat q^c_{\boldsymbol{m}}$ and $\hat q^s_{\boldsymbol{m}}$ in eq. (88) measure commuting, independent configuration-space coordinates. Hence eq. (88) describes ordinary change of coordinates in the configuration space from $\{q^{c,s}_{\boldsymbol{m}}\}$ to new coordinates $\{\phi(\boldsymbol{x_n})\}$. For a complete set of the new independent coordinates, it is convenient to take

$$\boldsymbol{x_n} \equiv \frac{L}{M}\,\boldsymbol{n}\,, \tag{91}$$

$\boldsymbol{n} = (n_1, n_2, n_3)$ with the integers $0 \le n_{1,2,3} < M$.

Also consider a Hermitian momentum field operator

$$\hat\pi(\boldsymbol{x}) \equiv \frac{1}{V^{1/2}}\sum_{\boldsymbol{m}} \sqrt{\frac{\omega_{\boldsymbol{m}}}{2}}\,(\hat p^c_{\boldsymbol{m}}\cos \boldsymbol{k_m}\!\cdot\!\boldsymbol{x}\; - \\ -\;\hat p^s_{\boldsymbol{m}}\sin \boldsymbol{k_m}\!\cdot\!\boldsymbol{x}) \tag{92}$$

where $\hat p^{c,s}_{\boldsymbol{m}} = -i\partial/\partial q^{c,s}_{\boldsymbol{m}}$ are the operators canonically conjugate to $\hat q^{c,s}_{\boldsymbol{m}}$. In terms of complex non-Hermitian operators

$$\hat q_{\boldsymbol{m}} \equiv \frac{1}{\sqrt 2}\,(\hat q^c_{\boldsymbol{m}} + i\hat q^s_{\boldsymbol{m}})\,, \quad \hat p_{\boldsymbol{m}} \equiv \frac{1}{\sqrt 2}\,(\hat p^c_{\boldsymbol{m}} + i\hat p^s_{\boldsymbol{m}}) \tag{93}$$

eqs. (88) and (92) read

$$\hat\phi(\boldsymbol{x}) = \frac{1}{V^{1/2}}\sum_{\boldsymbol{m}} \frac{1}{\sqrt{\omega_{\boldsymbol{m}}}}\,\hat q_{\boldsymbol{m}} e^{i\boldsymbol{k_m}\cdot\boldsymbol{x}}\,, \tag{94}$$

$$\hat\pi(\boldsymbol{x}) = \frac{1}{V^{1/2}}\sum_{\boldsymbol{m}} \sqrt{\omega_{\boldsymbol{m}}}\;\hat p_{\boldsymbol{m}} e^{i\boldsymbol{k_m}\cdot\boldsymbol{x}}\,. \tag{95}$$

For the last two equations we used the $\boldsymbol{m} \to -\boldsymbol{m}$ symmetry of the operators $q^{c,s}_{\boldsymbol{m}}$ and $p^{c,s}_{\boldsymbol{m}}$. For the complex operators (93) this symmetry yields

$$\hat q_{-\boldsymbol{m}} = \hat q^\dagger_{\boldsymbol{m}}\,, \quad \hat p_{-\boldsymbol{m}} = \hat p^\dagger_{\boldsymbol{m}}\,. \tag{96}$$

The operators $\hat q_{\boldsymbol{m}}$ and $\hat p_{\boldsymbol{m}}$ of eq. (93) satisfy the commutation relations

$$\begin{aligned} &[\hat q_{\boldsymbol{m}}, \hat p_{\boldsymbol{m}'}] = \delta_{\boldsymbol{m},-\boldsymbol{m}'}\,,\\ &[\hat q_{\boldsymbol{m}}, \hat q_{\boldsymbol{m}'}] = [\hat p_{\boldsymbol{m}}, \hat p_{\boldsymbol{m}'}] = 0\,. \end{aligned} \tag{97}$$

Using these commutators, we see that for $\boldsymbol{x_n}$ of eq. (91)

$$[\hat\phi(\boldsymbol{x_n}), \hat\pi(\boldsymbol{x_{n'}})] = \frac{i}{V}\,\delta_{\boldsymbol{n},\boldsymbol{n}'} \tag{98}$$

and for any two spatial points

$$[\hat\phi(\boldsymbol{x}), \hat\phi(\boldsymbol{x}')] = [\hat\pi(\boldsymbol{x}), \hat\pi(\boldsymbol{x}')] = 0\,. \tag{99}$$

[7] The terms with $\boldsymbol{m} = (0,0,0) \equiv \boldsymbol{0}$ require special treatment, especially for a massless field. Yet for a large volume V any observable effects of those terms are negligible. We thus simply disregard the $\boldsymbol{m} = \boldsymbol{0}$ terms and remove from the sum (88), which at this point is set at our will.

Let $\hat{F}(\boldsymbol{x})$ be an operator field that is composed from the operators $\hat{\phi}(\boldsymbol{x})$ and $\hat{\pi}(\boldsymbol{x})$. We identify

$$\int_V d^3x\,\hat{F}(\boldsymbol{x}) \equiv \left(\frac{L}{M}\right)^3 \sum_{\boldsymbol{n}} \hat{F}(\boldsymbol{x_n})\,. \tag{100}$$

We remember that here $L^3 = V$ and the sum has M^3 terms. Then eq. (98) is equivalent to the canonical field commutator

$$[\hat{\phi}(\boldsymbol{x}), \hat{\pi}(\boldsymbol{x}')] = i\delta^{(3)}(\boldsymbol{x}-\boldsymbol{x}')\,, \tag{101}$$

with the Dirac delta function defined for integral (100) by the usual prescription (15).

Consider the evolution transformation (83) that is generated by the Hamiltonian[8]

$$\hat{H} = \frac{1}{2}\sum_{\boldsymbol{m}} \frac{\omega_{\boldsymbol{m}}}{2} \sum_{\alpha=c,s} \left[(\hat{p}^\alpha)^2 + (\hat{q}^\alpha)^2 - 1\right] \tag{102}$$

The Gaussian wave function ψ_0 (82) is unchanged by this evolution. It is the ground (smallest-eigenvalue) eigenstate of the Hamiltonian (102).

We can introduce annihilation and creation operators

$$\begin{aligned} \hat{a}_{\boldsymbol{m}} &\equiv \frac{1}{\sqrt{2}}\left(\hat{q}_{\boldsymbol{m}} + i\hat{p}_{\boldsymbol{m}}\right), \\ \hat{a}^\dagger_{\boldsymbol{m}} &= \frac{1}{\sqrt{2}}\left(\hat{q}^\dagger_{\boldsymbol{m}} - i\hat{p}^\dagger_{\boldsymbol{m}}\right) = \frac{1}{\sqrt{2}}\left(\hat{q}_{-\boldsymbol{m}} - i\hat{p}_{-\boldsymbol{m}}\right). \end{aligned} \tag{103}$$

Then the Hamiltonian (102) equals

$$\hat{H} = \sum_{\boldsymbol{m}} \omega_{\boldsymbol{m}} a^\dagger_{\boldsymbol{m}} a_{\boldsymbol{m}} \tag{104}$$

provided that $\omega_{\boldsymbol{m}} = \omega_{-\boldsymbol{m}}$. By eq. (97),

$$\begin{aligned} [\hat{a}_{\boldsymbol{m}}, \hat{a}^\dagger_{\boldsymbol{m}'}] &= \delta_{\boldsymbol{m},\boldsymbol{m}'}\,, \\ [\hat{a}_{\boldsymbol{m}}, \hat{a}_{\boldsymbol{m}'}] &= [\hat{a}^\dagger_{\boldsymbol{m}}, \hat{a}^\dagger_{\boldsymbol{m}'}] = 0\,. \end{aligned} \tag{105}$$

For all $\boldsymbol{m}$,

$$\hat{a}_{\boldsymbol{m}}\psi_0 = 0\,. \tag{106}$$

The Hamiltonian (104) yields the standard evolution of the non-interacting theory. Specifically, for the annihilation operators in the Heisenberg representation:

$$\hat{a}_{\boldsymbol{m}}(t) = \hat{a}_{\boldsymbol{m}} e^{-i\omega_{\boldsymbol{m}} t}\,. \tag{107}$$

Expressing $\hat{q}_{\boldsymbol{m}}$ and $\hat{p}_{\boldsymbol{m}}$ in eqs. (94–95) through $\hat{a}_{\boldsymbol{m}}$ and $\hat{a}^\dagger_{\boldsymbol{m}}$, we find that at a time t

$$\hat{\phi}(t,\boldsymbol{x}) = \int d^3k\, \frac{1}{\sqrt{2\omega_{\boldsymbol{k}}}} \left(\hat{a}_{\boldsymbol{k}} e^{ik_\mu x^\mu} + \hat{a}^\dagger_{\boldsymbol{k}} e^{-ik_\mu x^\mu}\right), \tag{108}$$

$$\hat{\pi}(t,\boldsymbol{x}) = \int d^3k\, \sqrt{\frac{\omega_{\boldsymbol{k}}}{2}}\,(-i) \left(\hat{a}_{\boldsymbol{k}} e^{ik_\mu x^\mu} - \hat{a}^\dagger_{\boldsymbol{k}} e^{-ik_\mu x^\mu}\right). \tag{109}$$

Here $k_\mu \equiv (\omega_{\boldsymbol{k}}, \boldsymbol{k})$, we took $M \gg 1$, and we introduced a continuous operator field

$$\hat{a}_{\boldsymbol{k}} \equiv V^{1/2}\hat{a}_{\boldsymbol{m}(\boldsymbol{k})}\,, \tag{110}$$

with $\boldsymbol{m}(\boldsymbol{k}) \equiv [\boldsymbol{k}L/(2\pi)]$, where the brackets denote rounding off to the nearest integer. For the continuous annihilation operators (110)

$$\begin{aligned} [\hat{a}_{\boldsymbol{k}}, \hat{a}^\dagger_{\boldsymbol{k}'}] &= \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')\,, \\ [\hat{a}_{\boldsymbol{k}}, \hat{a}_{\boldsymbol{k}'}] &= [\hat{a}^\dagger_{\boldsymbol{k}}, \hat{a}^\dagger_{\boldsymbol{k}'}] = 0\,. \end{aligned} \tag{111}$$

We now specify the frequency parameters $\omega_{\boldsymbol{k}} \equiv \omega_{\boldsymbol{m}(\boldsymbol{k})}$. For any choice of $\omega_{\boldsymbol{k}}$, the fields (108–109) satisfy

$$\partial_t\hat{\phi} = \hat{\pi}\,. \tag{112}$$

The equation for $\partial_t\hat{\pi}$ will also be local in $\boldsymbol{x}$ if we set

$$\omega^2_{\boldsymbol{k}} = k^2 + \mu^2\,, \tag{113}$$

where $k^2 \equiv k_x^2 + k_y^2 + k_z^2$ and μ is a constant. Then

$$\partial_t\hat{\pi} = (\nabla^2 + \mu^2)\,\hat{\phi}\,, \tag{114}$$

where $\nabla^2 \equiv \partial_x^2 + \partial_y^2 + \partial_z^2$. The Hamiltonian (104) can in this case be expressed as the volume integral over local Hamiltonian density $\mathcal{H}(\hat{\phi}(\boldsymbol{x}), \hat{\pi}(\boldsymbol{x})) \equiv \hat{\mathcal{H}}(\boldsymbol{x})$:

$$\hat{H} \;=\; \int d^3x\, \hat{\mathcal{H}}(\boldsymbol{x})\,, \tag{115}$$

$$\hat{\mathcal{H}}(\boldsymbol{x}) \;=\; \frac{1}{2}\left[\hat{\pi}^2 + (\boldsymbol{\partial}\hat{\phi})^2 + \mu^2\hat{\phi}^2\right]. \tag{116}$$

Importantly, the field operators $\hat{\phi}(\boldsymbol{x})$ and $\hat{\pi}(\boldsymbol{x})$ evolve locally not only for the Hamiltonian above but also for *any local Hamiltonian* (115) with an arbitrary, possibly spacetime-dependent, function $\mathcal{H}(\hat{\phi}(x), \hat{\pi}(x), x)$. Of course, the Gaussian wave function ψ_0 then generally is no longer the ground state of the modified Hamiltonian.

Let us add to the free-field Hamiltonian density (116) other local operators multiplied by spacetime-dependent c-number coefficients. Let these coefficients start from zero and change adiabatically to non-zero values over a characteristic timescale T. Under the evolution by the new Hamiltonian all the modes with frequency $\omega \gg T^{-1}$ will remain in the ground state. This shows that in order to restrict the Hamiltonian to a specific form, it is insufficient to require the high-frequency modes to be in the ground state and the evolution to be local.

The quantum non-interacting scalar field (88) could be said in certain sense to exist within the generic structure $\nu(q)$, ubiquitously encountered in our surroundings. Yet this "emergent" quantum system cannot be an objectively existing physical world because its evolution by the Hamiltonian (115–116) does not stand out from nearby paths of its continuous evolution with arbitrarily modified Hamiltonians, local or non-local. In what follows

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[8] An additional factor 1/2 in eq. (102) corrects for counting every cosine and sign mode twice, at $\boldsymbol{m}$ and $-\boldsymbol{m}$.

we identify other ubiquitously present emergent quantum field systems that do not blend with other, continuously modified systems. Thus they are objective distinct physical entities. These systems will possess such characteristics of the world of ours as gauge and gravitational interactions.

V. GAUGE FIELDS

A notable feature of the observed physical dynamics is the high degree of its symmetry. In particular, the dynamical field equations are invariant under vast groups of local gauge and diffeomorphism transformations.

We continue to consider the Hilbert space that is constructed as described above from the linear combinations (54) that smoothly fit the generic structure $\nu(q)$. On this Hilbert space we will identify states and field operators with the gauge and diffeomorphism symmetries. In this section we discuss the gauge symmetry and in the next Sec. VI the diffeomorphism symmetry. In Sec. VIII we show that the physical dynamics of emergent fields with sufficient local symmetry, in particular local supersymmetry, is unambiguous. These emergent systems thus represent distinct worlds with definite physical laws. This suggests that of the various theoretically conceivable quantum field theories only those with enough symmetry are generically realized in collections of basic objects as worlds suitable for developing intelligent life.

A. Gauge fields as degrees of freedom

We straightforwardly generalize the construction of a free field $\hat{\phi}(x)$ and its conjugate momentum field $\hat{\pi}(x)$ in previous Sec. IV D to a pair of evolving free fields $\hat{\phi}^\alpha(x)$, with $\alpha = 1, 2$, and the conjugate momenta $\hat{\pi}_\alpha(x)$. To this end we simply substitute each of the operators $\hat{q}_{\boldsymbol{m}}^{c(\text{or } s)}$ in eq. (88) by two independent operators $\hat{q}_{\boldsymbol{m}}^{c(\text{or } s)\alpha}$ that measure various uncorrelated coordinates q^n of the basic structure $\nu(q)$. We then proceed with the rest of the construction for the two fields in full analogy with Sec. IV D.

For the pair of fields we may generalize the Hamiltonian density (116) to

$$\mathcal{H} = \sum_\alpha \frac{1}{2}\left[\hat{\pi}_\alpha^2 + (\boldsymbol{\partial}\hat{\phi}^\alpha)^2 + \mu^2(\hat{\phi}^\alpha)^2\right], \tag{117}$$

where we set equal mass to both components ϕ^α. As noted earlier, we can as well consider another path of evolution of the wave function or, equivalently, of the Heisenberg field operators. If such alternative evolution is generated by a Hamiltonian that is a spatial integral of a local function of the fields, $\int d^3x\ \mathcal{H}(\hat{\phi}^\alpha(x), \hat{\pi}_\alpha(x), x)$, then the field dynamics remains local.

As a possible modification of the Hamiltonian density (117), let us consider

$$\mathcal{H} = \sum_\alpha \frac{1}{2}\left[\hat{\pi}_\alpha^2 + (\boldsymbol{D}\hat{\phi}^\alpha)^2 + \mu^2(\hat{\phi}^\alpha)^2\right] \tag{118}$$

where

$$\boldsymbol{D}\hat{\phi}^\alpha \equiv \boldsymbol{\partial}\hat{\phi}^\alpha - i^\alpha_\beta \boldsymbol{A}(x)\hat{\phi}^\beta \tag{119}$$

and i is the 2×2 matrix (62). So far $\boldsymbol{A}(x)$ is an arbitrary three-component function on spacetime. It is not a dynamical field operator yet.

The Hamiltonian density (118) is invariant under local gauge transformation with a time-independent phase change $\varphi(\boldsymbol{x})$:

$$\hat{\phi} \equiv \begin{pmatrix} \hat{\phi}^1 \\ \hat{\phi}^2 \end{pmatrix} \ \rightarrow\ \hat{\phi}_\varphi = e^{i\varphi(\boldsymbol{x})}\hat{\phi}\,, \tag{120a}$$

$$\hat{\pi} \equiv (\hat{\pi}_1, \hat{\pi}_2)\ \rightarrow\ \hat{\pi}_\varphi = \hat{\pi}e^{-i\varphi(\boldsymbol{x})}\,, \tag{120b}$$

$$\boldsymbol{A}\ \rightarrow\ \boldsymbol{A}_\varphi = \boldsymbol{A} + \boldsymbol{\partial}\varphi\,. \tag{120c}$$

Transformation (120a, 120b) is canonical for $\hat{\phi}$ and $\hat{\pi}$. It can be obtained through the similarity transformation (23) with an operator

$$\hat{U}_\varphi = e^{-i\int d^3x\, \varphi(\boldsymbol{x})\hat{j}^0(\boldsymbol{x})} \tag{121}$$

where

$$\hat{j}^0 = \sum_\alpha \frac{\partial\hat{\phi}^\alpha_\varphi}{\partial\varphi}\,\hat{\pi}_{\varphi\,\alpha} = \sum_\alpha (\hat{i}\hat{\phi})^\alpha\hat{\pi}_\alpha\,. \tag{122}$$

The gauge-rotated field $\hat{\phi}_\varphi(\boldsymbol{x})$ describes the same physical system in different coordinates of the configuration space. The wave function $\psi(\phi)$ in the old coordinates $\phi(\boldsymbol{x})$ is related to the wave function $\psi_\varphi(\phi_\varphi)$ in the new coordinates $\phi_\varphi(\boldsymbol{x})$ by a transformation

$$\psi\ \rightarrow\ \psi_\varphi = \hat{U}_\varphi\psi\,. \tag{123}$$

Evolution that in the old coordinates is specified by the Hamiltonian density (118–119), in the new coordinates is generated by the same Hamiltonian density but with the gauge connection field $\boldsymbol{A}(x)$ in eq. (119) adjusted according to eq. (120c).

Next we identify another quantum system that contains additional physical degrees of freedom but is likewise represented by the same smooth fitting functions (39) for the generic basic distribution. We start from the considered earlier emergent wave function $\psi(\phi^1, \phi^2)$. It appeared as a description of the smooth function $\rho(q^{c\alpha}_{\boldsymbol{m}}, q^{s\alpha}_{\boldsymbol{m}})$ that fitted the basic structure $\nu(q)$. We then evolve the wave function $\psi(\phi^1, \phi^2)$ by the Hamiltonian (118–119) where now we let the

field $\boldsymbol{A}(x)$ vary with other independent coordinates q^n of the same generic basic structure $\nu(q)$.

Similarly to eq. (88), we match some of the additional coordinates q^n to the transverse (gauge-invariant) modes of the connection field $\boldsymbol{A} = \boldsymbol{A}^T + \boldsymbol{\partial}\varphi$, with $\boldsymbol{\partial}\cdot\boldsymbol{A}^T = 0$:

$$\boldsymbol{A}^T(\boldsymbol{x}) = \frac{1}{V^{1/2}} \sum_{\boldsymbol{m},\lambda} \frac{\boldsymbol{\epsilon}_{\boldsymbol{m}\lambda}}{\sqrt{2\omega_{\boldsymbol{m}}}} \left[q^{c\lambda}_{\boldsymbol{m}}(q) \cos \boldsymbol{k}_{\boldsymbol{m}}\cdot\boldsymbol{x} \ + \right. \\ \left. + \ q^{s\lambda}_{\boldsymbol{m}}(q) \sin \boldsymbol{k}_{\boldsymbol{m}}\cdot\boldsymbol{x}\right]. \quad (124)$$

Here $\lambda \in \{1, 2\}$, $\boldsymbol{\epsilon}_{\boldsymbol{m}\lambda}\cdot \boldsymbol{\epsilon}_{\boldsymbol{m}\lambda'} = \delta_{\lambda\lambda'}$, $\boldsymbol{k}_{\boldsymbol{m}}\cdot \boldsymbol{\epsilon}_{\boldsymbol{m}\lambda} = 0$, and $q^{c(\text{or } s)\lambda}_{\boldsymbol{m}}(q)$ are some linearly-independent functions. Now any value of q specifies a scalar field configuration $\phi(\boldsymbol{x})$ (88) and a transverse field configuration $\boldsymbol{A}^T(\boldsymbol{x})$ (124).

We do not promote for a physical degree of freedom the longitudinal (gauge-dependent) part $\boldsymbol{\partial}\varphi$ of $\boldsymbol{A}(\boldsymbol{x})$, i.e., the connection field modes with polarization $\boldsymbol{\epsilon}_{\boldsymbol{m}3} = \boldsymbol{k}_{\boldsymbol{m}}/|\boldsymbol{k}_{\boldsymbol{m}}|$. Instead we regard gauge-equivalent configurations (120) as describing the same physical system in various coordinate frames of the configuration space. We then can impose any longitudinal potential $\varphi(\boldsymbol{x})$, indicating the frame used.

In the frame that corresponds to the "radiation gauge" $\boldsymbol{\partial}\cdot\boldsymbol{A} = 0$ we have $\boldsymbol{A} = \boldsymbol{A}^T$. Setting this gauge, let us determine an emergent wave function $\psi(\phi, \boldsymbol{A}^T)$, or equivalently $\psi(q^{c\alpha}_{\boldsymbol{m}}, q^{s\alpha}_{\boldsymbol{m}}, q^{c\lambda}_{\boldsymbol{m}}, q^{s\lambda}_{\boldsymbol{m}})$. In the preceding sections the dependence of the wave function on the scalar field modes $q^{c\alpha}_{\boldsymbol{m}}$ and $q^{s\alpha}_{\boldsymbol{m}}$ followed from the assignment (53). It brought the physically motivated scalar product to the standard form (52). Accepting for now this form of the wave function for its scalar field argument, we are still seemingly free to match the modes of the connection field (124) to any functions $q^{c\lambda}_{\boldsymbol{m}}(q)$ and $q^{s\lambda}_{\boldsymbol{m}}(q)$ of the basic coordinates q of the underlying structure.

Moreover, given a structure $\nu(q)$, we can consider an ensemble $\mathcal{V} \equiv (\nu, \nu, \ldots)$. In it we may arbitrarily impose its own connection $\boldsymbol{A}^T(\boldsymbol{x})$ on every member of this ensemble. In the ensemble $\mathcal{V}$ the coordinates $q^{c\lambda}_{\boldsymbol{m}}$ and $q^{s\lambda}_{\boldsymbol{m}}$ extend along the dimensions that we have "created" by considering the multiple copies of the fundamental structure $\nu(q)$. Then even if the dependence of the wave function on ϕ could somehow be fixed, its dependence on $\boldsymbol{A}^T$ can still be set arbitrarily by our choice.

If, by nature of $\nu(q)$ objects, their properties q range over a finite set of discrete values then the choice of distinct evolution flows, characterized by different $\boldsymbol{A}^T$, can also be finite. If so, we could contemplate matching the physical world to the ensemble that contains all the distinct possibilities for the evolution. However, first, it is unclear if the resulting dependence of ψ on $\boldsymbol{A}^T$ would then be of any generic form, insensitive to the allowed values of the objects' properties q (in some natural coordinates). Second, we will see in Sec. VII E that the physical world that develops through inflationary expansion can evolve from only a specific wave function for the high-energy modes.

Therefore, it does not matter whether or not the set of all the evolution flows has a generic form. In either case, the physical states that resemble or represent our world evolve from only a special ensemble of evolution flows. Indistinguishably, such antropically and phenomenologically acceptable states evolve from only certain special matching of $\boldsymbol{A}^T$ modes to the independent coordinates q^n in eq. (124).[9]

B. Wave function is constant on gauge orbits

In the standard axiomatically formulated gauge theories not only the action but also the wave function is necessarily gauge invariant. The gauge symmetry of the wave function is required for theory consistency. It constitutes the so-called "secondary constraint." We demonstrate this explicitly in Appendix A, eq. (A16), for locally Lorentz-invariant renormalizable theories with arbitrary abelian or non-abelian gauge symmetry.

Let us investigate how a wave function can be gauge invariant in a theory that is a low-resolution description of the generic basic structure $\nu(q)$. In the radiation gauge ($\boldsymbol{\partial}\cdot\boldsymbol{A} = 0$), regarded by Sec. V A as a certain coordinate frame in the configuration space, we describe the emergent physical system by a wave function $\psi(\phi, \boldsymbol{A}^T)$ as discussed in the previous subsection. In another gauge (frame of the configuration space) with

$$\boldsymbol{A} = \boldsymbol{A}^T + \boldsymbol{\partial}\varphi \quad (125)$$

the scalar field is measured by the operator $\hat{\phi}_\varphi$ of eq. (120a). In this frame we use the wave function $\psi(\phi_\varphi, \boldsymbol{A})$, the dynamical variables of which are the transformed scalar field ϕ_φ and the gauge-invariant transverse modes of $\boldsymbol{A}$, given by eq. (124). The wave functions $\psi(\phi, \boldsymbol{A}^T)$ and $\psi(\phi_\varphi, \boldsymbol{A})$ are the projections of the same state on two unitarily-equivalent sets of basis vectors in the Hilbert space. These wave functions are related by the linear transformation (123) with $\hat{U}_\varphi$ of eqs. (121–122). We rewrite this $\hat{U}_\varphi$ as

$$\hat{U}_\varphi = \exp\left\{-\int d^3x\, \varphi(\boldsymbol{x})\, (\hat{i}\hat{\phi})^\alpha \frac{\delta}{\delta\phi^\alpha(\boldsymbol{x})}\right\}. \quad (126)$$

Then, by the above,

$$\psi(\phi, \boldsymbol{A}) = \hat{U}_\varphi \psi(\phi, \boldsymbol{A}^T) = \psi(e^{-\hat{i}\varphi}\phi, \boldsymbol{A}^T). \quad (127)$$

[9] Incidentally, we could make the same arguments about the dependence of ψ on the scalar field ϕ. We will return to this point in Sec. VII E. We will find that the "generic" Gaussian wave function ψ_0 (82) indeed does not need to describe the high-energy modes of the scalar field. However, it will appear most appropriate to regard the inflationary-suitable wave function of the matter fields as a certain transformation of ψ_0.

Equivalently,

$$\psi(e^{\hat{i}\varphi}\phi, \boldsymbol{A}^T + \boldsymbol{\partial}\varphi) = \psi(\phi, \boldsymbol{A}^T)\,. \tag{128}$$

Thus the considered wave function $\psi(\phi, \boldsymbol{A})$ has the same value at every point $(\phi, \boldsymbol{A})$ of an orbit of the gauge group (120).

Differentiation of both sides of eq. (128) over φ yields

$$(\hat{j}^0 - \partial_i \hat{\pi}^i)\,\psi = 0\,, \tag{129}$$

where

$$\hat{j}^0(\boldsymbol{x}) = -i(\hat{i}\phi)^\alpha \frac{\delta}{\delta\phi^\alpha(\boldsymbol{x})} \tag{130}$$

is the operator of current density (122), and $\hat{\pi}^i(\boldsymbol{x})$ are the operators of the momenta conjugate to $A_i(\boldsymbol{x})$,

$$\hat{\pi}^i\,\psi(\phi, \boldsymbol{A}) \equiv -i\frac{\delta}{\delta A_i(\boldsymbol{x})}\,\psi(\phi, \boldsymbol{A}). \tag{131}$$

The operator $\hat{j}^0 - \partial_i\hat{\pi}^i$ of eq. (129) generates the gauge transformation (120). Indeed, let $\hat{O}$ be any of the operators $\hat{\phi}^\alpha$, $\hat{\pi}_\alpha$, $\hat{\boldsymbol{A}}$, or their arbitrary function. If $\hat{O}_\varphi$ is the result of $\hat{O}$ gauge transformation (120) then

$$\frac{\delta \hat{O}_\varphi}{\delta\varphi(\boldsymbol{x})} = [\hat{j}^0 - \partial_i\hat{\pi}^i(\boldsymbol{x}), \hat{O}_\varphi]\,. \tag{132}$$

Therefore, eq. (129) can be used interchangeably with eq. (128) to express the constancy of the wave function along the gauge orbits.

C. Four-vector gauge field and Hamiltonian

We continue to study an emergent quantum system whose dynamical variables are the scalar fields $\phi^\alpha(\boldsymbol{x})$ and the transverse components of the vector connection field $\boldsymbol{A}(\boldsymbol{x})$. For this system we need to extend the scalar-field Hamiltonian density (118) to one that also specifies the evolution of the dynamical degrees of freedom $\boldsymbol{A}^T$. Every such extension generates a continuous group of evolution transformations of the wave function (or of the Heisenberg field operators). By reasons to be explained in full in Sec. VIII, we focus attention on the following narrow class of these choices. Foremost, we consider only the evolution under which the scalar fields $\hat{\phi}^\alpha(\boldsymbol{x})$ and their conjugate momenta $\hat{\pi}_\alpha(\boldsymbol{x})$ evolve locally in space $\{\boldsymbol{x}\}$. We need this because the gauge connection $\boldsymbol{A}$ exists only in relation to such local evolution, generated by a Hamiltonian density such as (118). Likewise, we explore only local evolution of the connection $\hat{\boldsymbol{A}}(\boldsymbol{x})$ because of anticipating other, gravitational physical degrees of freedom whose existence is tied to the locality of evolution of all the elementary fields, Sec. VI.

Thus our Hamiltonian is a spatial integral of a Hamiltonian density $\hat{\mathcal{H}}(\boldsymbol{x})$ that is a function of the field and momentum operators at $\boldsymbol{x}$ or of their spatial derivatives at $\boldsymbol{x}$. We explore only the Hamiltonians that in addition to being local and gauge-invariant are renormalizable. The requirement of renormalizability for the matter and gauge fields well below the Planck energy scale stems from the usual considerations that the renormalizable theories are a generic low-energy limit of arbitrary field theories [37]. Finally, we require that for the Minkowski metric, used so far, the field operators evolve by Lorentz-invariant equations. We will justify local Lorentz invariance in Sec. VIII.

Let $\hat{\mathcal{H}} = \mathcal{H}(\hat{\phi}^\alpha, \hat{\pi}_\alpha, \hat{A}_i, \hat{\pi}^i)$ be some Hamiltonian density that satisfies the stated requirements. Then, for any function $A_0(x)$, another Hamiltonian density

$$\hat{\mathcal{H}}' \equiv \hat{\mathcal{H}} + A_0(x)\,(\hat{j}^0 - \partial_i\hat{\pi}^i) \tag{133}$$

yields the same Schrodinger evolution of the wave function due to eq. (129):

$$d\psi = -idt\hat{H}'\psi = -idt\hat{H}\psi\,. \tag{134}$$

Eq. (132) shows that the additional transformation

$$-\,idt(\hat{H}' - \hat{H})\psi = -idt\int d^3x\,A_0\,(\hat{j}^0 - \partial_i\hat{\pi}^i)\,\psi$$

is the gauge transformation with

$$d\varphi(x) = dt\,A_0(x)\,. \tag{135}$$

By eq. (129), the gauge transformation does not affect the wave function ψ, but it does change field operators. The Heisenberg operator $\hat{\phi}(t, \boldsymbol{x})$ evolves under the modified Hamiltonian $\hat{H}'$ as

$$\frac{\partial\hat{\phi}}{\partial t} = i[\hat{H}', \hat{\phi}] = i[\hat{H}, \hat{\phi}] + iA_0\hat{\phi}\,. \tag{136}$$

Introducing a gauge-covariant 4-derivative

$$D_\mu = \partial_\mu - \hat{i}A_\mu\,, \tag{137}$$

we rewrite eq. (136) as

$$D_0\hat{\phi} = i[\hat{H}, \hat{\phi}]\,. \tag{138}$$

Thus the c-number field $A_0(x)$ is indeed the temporal component of the 4-vector connection $A_\mu = (A_0, \boldsymbol{A})$. The gauge-covariant eq. (138) describes evolution of the field $\hat{\phi}$ in any gauge, with an arbitrary $\varphi(t, \boldsymbol{x})$.

The gauge transformation of A_0,

$$A_0 \to A_0 + \partial_0\varphi\,, \tag{139}$$

follows from eq. (135). Indeed, the field operators at t that are gauge-transformed by $\varphi(t)$ and at $t + dt$ that are transformed by $\varphi(t + dt)$ become connected through eq. (135) by A_0 that is adjusted by eq. (139).

Since for the evolution by $\hat{H}'$

$$\frac{\partial\hat{A}_i}{\partial t} = i[\hat{H}', \hat{A}_i] = i[\hat{H}, \hat{A}_i] + \hat{i}A_0\,, \tag{140}$$

we also have

$$\hat{F}_{0i} = i[\hat{H}, \hat{A}_i]\,, \tag{141}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the gauge-invariant field-strength tensor of the considered abelian gauge theory.

It is much easier to explore the possible laws of Lorentz-invariant canonical evolution in the Lagrangian description. A local Hamiltonian (115) leads to a local action $S = \int d^4x\,\mathcal{L}(x)$. This action gives Lorentz-covariant field equations, e.g. [38], whenever the Lagrangian density $\mathcal{L}$ is a 4-scalar function of the fields ϕ and A_μ, or of their spacetime derivatives.

In the presented view of quantum evolution as a sequence of alternative linear representations of a fitting function (54), the action formalism corresponds to the description of evolution by the convolution (28). It specifies the Schrodinger evolution of a wave function by the Feynman path integral, eq. (31). When the Hamiltonian is at most quadratic in the momenta fields then in eq. (31) we explicitly integrate over $d\!\!\!{}^-\pi$ with the result

$$\psi'(f') = \int [df]\, e^{i\int d^3x\,\mathcal{L}(\dot{f},f)dt}\;\psi(f)\,. \tag{142}$$

Here $f = \{f^\alpha\}$ denotes all the dynamical fields, $\dot{f} = (f' - f)/dt$, and the path-integral measure $[df]$, as usually, absorbs the constants from the integration in eq. (31) over the independent momenta $\pi_{\alpha\boldsymbol{n}} \equiv \pi_\alpha(\boldsymbol{x}_{\boldsymbol{n}})$ [cf. eq. (100)].

In Appendix B we explicitly write the Hamiltonian that follows from the general locally Lorentz-invariant, renormalizable, local action of bosonic fields with an arbitrary gauge symmetry, abelian or non-abelian. We then map the basic coordinates q^n to the modes of the scalar and gauge fields ϕ^α and A_i^a modulus their joint gauge transformation similarly to the earlier case of the abelian theory in flat spacetime. Transformation

$$\psi \to e^{-i(\hat{H}^\phi + \hat{H}^A)dt}\,\psi \tag{143}$$

with the Hamiltonian of eqs. (B3–B10) gives us the Schrodinger wave function of the general abelian or non-abelian gauge field theory at various moments of time.

VI. GENERAL COVARIANCE AND QUANTUM GRAVITY

We continue to analyze quantum field systems that exist as a low-resolution representation of the commonly encountered static structure $\nu(q)$. In this section we consider emergent field systems that are symmetric under diffeomorphism transformations. The latter canonically transform Heisenberg field operators $\hat{f}(x)$, where $x \equiv (t, \boldsymbol{x})$, to

$$\hat{f}'(x) = \hat{U}_\varepsilon^{-1}\hat{f}(x)\,\hat{U}_\varepsilon\,. \tag{144}$$

For any infinitesimal 4-vector "displacement" parameter field $\varepsilon^\mu(x)$, the change $\delta_\varepsilon \hat{f} \equiv \hat{f}'(x) - \hat{f}(x)$ under the diffeomorphism transformation of a scalar, vector, or higher-tensor field is by definition the field Lie derivative along ε^μ. For example,

$$\begin{aligned}
&\delta_\varepsilon\hat{\phi} = L_\varepsilon\hat{\phi} = \varepsilon^\lambda\hat{\phi}_{,\lambda}\\
&\delta_\varepsilon\hat{A}_\mu = L_\varepsilon\hat{A}_\mu = \varepsilon^\lambda\hat{A}_{\mu,\lambda} + \varepsilon^\lambda_{,\mu}\hat{A}_\lambda\\
&\delta_\varepsilon\hat{g}_{\mu\nu} = L_\varepsilon\hat{g}_{\mu\nu} = \varepsilon^\lambda\hat{g}_{\mu\nu,\lambda} + \varepsilon^\lambda_{,\mu}\hat{g}_{\lambda\nu} + \varepsilon^\lambda_{,\nu}\hat{g}_{\mu\lambda}\,.
\end{aligned} \tag{145}$$

The transformed field operator $\hat{f}'(x)$ describes the physical system in a new spacetime coordinate frame, shifted relative to the old one by $\Delta x^\mu = \varepsilon^\mu(x)$. In the new frame

$$\hat{\phi}'(x') = \hat{\phi}(x), \quad \hat{A}'_\mu(x') = \frac{\partial x^\nu}{\partial x'^\mu}\,\hat{A}_\nu(x), \quad \ldots \tag{146}$$

with

$$x' = x - \varepsilon. \tag{147}$$

We will confirm that the Hamiltonian of a diffeomorphism-invariant (generally covariant) physical system should be in a special form (157). For such a system it will be possible to identify a field operator with the interpretation of a spatial metric tensor. Invariance to *spatial* diffeomorphism transformations, forming a subgroup of the spacetime diffeomorphism group (145), requires the constancy of the wave function on orbits of spatial diffeomorphism. Thus, similarly to the gauge symmetry, the independent coordinates q^n of the basic structure should now be mapped to the entire orbits of the spatial diffeomorphism group. This section and Appendix B explicitly demonstrate the existence of scalar, gauge vector, and gravitational tensor quantum fields that evolve by generally covariant laws and are represented by fitting functions for the generic basic distribution $\nu(q)$. The question of why general covariance applies to the physical world that we live in will be addressed in further Sec. VIII.

A. Hamiltonian for general covariance

To find emergent quantum systems with diffeomorphism-invariant evolution we evoke an argument presented by Dirac [39] for the Hamiltonian formulation of classical general relativity. We use the more modern ADM notation (5) [34, 35]. We also remember that the dynamical variables in the considered here quantum case are described by operators.

Let $\hat{\eta}$ be an operator for a prospective observable that is a function of the dynamical fields $\hat{f}^\alpha(x)$ at a common time x^0. The function

$$\hat{\eta} \equiv \eta(\hat{f}^\alpha) \tag{148}$$

may or may not be localized at one spatial point $\boldsymbol{x}$. For the infinitesimal diffeomorphism transformation with a

displacement parameter field $\varepsilon^\mu(x)$ we have

$$\delta\hat{\eta} = \int d^3x\, \varepsilon^\mu(\boldsymbol{x})\, \hat{\xi}_\mu(\boldsymbol{x}), \tag{149}$$

where $\hat{\xi}_\mu$ do not depend on ε^μ. We require that the operator $\hat{U}_\varepsilon$ of eq. (144) preserves the canonical form of the Hermitian product (79) and is therefore unitary. Then for an infinitesimal ε^μ we should take

$$\hat{U}_\varepsilon = e^{-i\int d^3x\, \varepsilon^\mu(\boldsymbol{x})\, \hat{\mathcal{H}}_\mu(\boldsymbol{x})} \tag{150}$$

where $\hat{\mathcal{H}}_\mu$ are Hermitian operators that are independent of ε^μ. Comparing eqs. (144, 150) and eq. (149) we see that

$$\frac{\delta\hat{\eta}}{\delta\varepsilon^\mu(\boldsymbol{x})} = \hat{\xi}_\mu(\boldsymbol{x}) = i[\hat{\mathcal{H}}_\mu(\boldsymbol{x}), \eta]\,. \tag{151}$$

Similarly to the "normal" evolution with $A^a_0 = 0$ for gauge-symmetric systems in eq. (A13), we set a normal unit temporal displacement field $n^\mu(\boldsymbol{x})$. In close analogy to the gauge symmetry, we can then use symmetry considerations to extend the evolution along the normal direction $n^\mu(\boldsymbol{x})$ to diffeomorphism-equivalent evolution for another displacement $\varepsilon^\mu(\boldsymbol{x})$.

To this end we decompose the arbitrary displacement vector $\varepsilon^\mu(\boldsymbol{x})$ into normal and tangential components as

$$\varepsilon^\mu = [Cn^\mu + (0, \boldsymbol{C})]\, dt\,, \tag{152}$$

where the normalization of the vector in the brackets is arbitrary. We extend our spatial coordinates $\{\boldsymbol{x}\}$ to other times $t \neq x^0$ along the vector field $\varepsilon^\mu(\boldsymbol{x})$, and we set the time coordinate as $t \equiv x^0 + dt$, with dt from eq. (152). In the introduced spacetime coordinates

$$\varepsilon^\mu(\boldsymbol{x}) = (1, \boldsymbol{0})\, dt\,. \tag{153}$$

In a generally covariant theory we should be able to provide a normal unit vector n^μ for every spatial hypersurface. Therefore, we should have the concept of the *metric tensor*. Let the metric tensor in the coordinate frame (153) be parameterized by the ADM lapse $N(\boldsymbol{x})$ and shift $N^i(\boldsymbol{x})$, as given by eq. (5). The unit vector n^μ, normal in the metric (5) to the $dt = 0$ hypersurface, then equals

$$n_\mu = (-N, \boldsymbol{0})\,, \quad n^\mu = \left(\frac{1}{N}, -\frac{N^i}{N}\right). \tag{154}$$

Hence the displacement ε^μ of eq. (152) takes the form (153) when $C = N$ and $C^i = N^i$, giving

$$\varepsilon^\mu = [Nn^\mu + (0, N^i)]\, dt = (Nn^0, Nn^i + N^i)\, dt. \tag{155}$$

Then by eqs. (149,155,151),

$$\delta\hat{\eta} = i[\hat{H}, \hat{\eta}]\, dt \tag{156}$$

with

$$\begin{aligned}\hat{H} &= \int d^3x \left(N\, \hat{\mathcal{H}}_N + N^i \hat{\mathcal{H}}_i\right) \equiv \\ &\equiv \int d^3x\; N^\alpha \hat{\mathcal{H}}_\alpha\,,\end{aligned} \tag{157}$$

where $\hat{\mathcal{H}}_N = n^\mu \hat{\mathcal{H}}_\mu$, $N^\alpha \equiv (N, N^i)$, and $\hat{\mathcal{H}}_\alpha \equiv (\hat{\mathcal{H}}_N, \hat{\mathcal{H}}_i)$. (In these notations $\mathcal{H}_{\mu=0}$ generally differs from $\mathcal{H}_{\alpha=0} \equiv \hat{\mathcal{H}}_N$. The former is the temporal component of the Hamiltonian density and the latter is its normal component.)

B. Spatial diffeomorphism invariance

The observed physical evolution is generally covariant; therefore, it is also 3-diffeomorphism invariant. In other words, its laws are invariant under a general change of *spatial* coordinates $\boldsymbol{x} \to \boldsymbol{x} - \boldsymbol{\varepsilon}$, with simultaneous local rotation of the fields according to their 3-tensor rank. It infinitesimally changes the dynamical fields by their Lie derivative along the spatial displacement field $\boldsymbol{\varepsilon}(\boldsymbol{x})$, i.e., by the 3-dimensional equivalent of eq. (145).

The diffeomorphism invariance (for either 3 or 4 dimensions) could be achieved directly by introducing the affine connections and promoting them to quantum operators. This would lead us to the Einstein-Cartan theory [40]. However, we can reduce the number of new independent geometrical degrees of freedom by requiring the Einstein equivalence principle. Under it, the connections $\Gamma^\lambda_{\mu\nu}$ are metric-compatible ($g_{\mu\nu;\lambda} = 0$) and symmetric. It is then sufficient to follow the machinery of general relativity and use only our metric tensor to construct the symmetric Christoffel symbols and an invariant action, yielding the required Hamiltonian. This procedure fails to incorporate covariantly half-integer spins, to be discussed in a later paper about fermions. But for the bosonic integer-spin fields, studied here, we build a generally covariant theory where all the independent geometrical degrees of freedom are only certain components of the metric tensor.

We thus implement 3-covariance by introducing new, geometrical, degrees of freedom that are described by a 3-metric tensor field operator $\hat{\gamma}_{ij}(\boldsymbol{x})$. We add these degrees of freedom in full analogy with introduction of the gauge degrees of freedom $\hat{A}_i$ in Sec. V. Namely, we consider various evolution paths of the matter fields, here $(\hat{\phi}, \hat{A}_i)$, that are generated by Hamiltonians $H^m(\hat{\phi}, \hat{A}_i, \hat{\pi}_\phi, \hat{\pi}_{A_i}, \gamma_{ij})$ for the matter. The last argument, $\gamma_{ij}(x)$, is here a parameter field that by definition enters H^m as expected for the spatial metric in a Hamiltonian obtained from a diffeomorphism-invariant action, cf. Appendix B. Then, as shown next, similarly to the gauge symmetry, we map some components of $\gamma_{ij}(\boldsymbol{x})$ to "unused" independent coordinates q^n of the basic structure $\nu(q)$ (or of the corresponding ensemble $\mathcal{V}$, Sec. V A). These independent coordinates will then represent the geometrical degrees of freedom.

C. Spacetime diffeomorphism invariance

In a generally covariant theory, at a given fixed time, the same wave function should specify the initial state for evolution with every conceivable lapse and shift N^α of the displacement (155). Therefore, the wave function should be independent of N^α. The change of the wave function due to evolution, $-idt\hat{H}\psi$, should not introduce the dependence on N^α either. This yields the Hamiltonian and momentum constraints [32] of quantum gravity:

$$\hat{\mathcal{H}}_\alpha \psi = 0\,. \tag{158}$$

As seen from eq. (B15), the momentum constraints, $\hat{\mathcal{H}}_i\psi = 0$, express constancy of the wave function $\psi(\phi, A_i, \gamma_{ij})$ on the orbits of 3-diffeomorphism transformations of ϕ, A_i and γ_{ij}, Ref. [32, 41]. We can use these symmetry transformations to remove 3 of the 6 independent components of the symmetric tensor γ_{ij}, for example, as follows. Spatial diffeomorphism transformation changes the 3-metric tensor as

$$\delta_{\boldsymbol{\varepsilon}}\gamma_{ij} = L_{\boldsymbol{\varepsilon}}\gamma_{ij} = \varepsilon_{(i|j)}\,, \tag{159}$$

where $\varepsilon_i = \gamma_{ij}\varepsilon^j$, $|$ indicates 3-covariant derivative, and parentheses denote symmetrization. We then determine 3 functions $\varepsilon_i(\boldsymbol{x})$ for which the transformation (159) brings 3 components of γ_{ij} to some standard form (amounting to a gauge condition). By the momentum constraints, this does not affect the wave function ψ.

We now turn attention to the Hamiltonian constraint, $\hat{\mathcal{H}}_N\psi = 0$. Let us define a scale factor $a(\boldsymbol{x})$ by

$$\gamma_{ij} \equiv a^2 h_{ij}\,, \quad \det h_{ij} \equiv 1\,. \tag{160}$$

The Hamiltonian constraint relates the values of ψ at different $a(\boldsymbol{x})$ [cf. the Wheeler-DeWitt equation (174) below]. Then only 2 components of γ_{ij} remain independent and need to be mapped to the basic coordinates q similarly to eq. (124). Thus we can regard $a(\boldsymbol{x})$ as another arbitrary transformation parameter, analogous to $\varphi(\boldsymbol{x})$ and $\varepsilon^i(\boldsymbol{x})$.

Quantitative exploration of generally covariant dynamics is much simpler in the Lagrangian formulation of the theory. Consider a local gauge-invariant and diffeomorphism-invariant action [for $m_P^2 \equiv (8\pi G)^{-1} \equiv 1$]

$$S \equiv \int d^4x \left[\frac{\sqrt{-g}}{2}\, R - \sum_s \frac{\sqrt{-g}}{4e_s^2}\, F^{sa}_{\mu\nu}F^{sa\,\mu\nu} + {} + \mathcal{L}^\phi(D_\mu\phi, \phi, g_{\mu\nu})\right], \tag{161}$$

where R is the Riemann curvature scalar, e_s^2 are the gauge couplings (possibly different for different simple subgroups s of the overall gauge group), and $S^\phi \equiv \int d^4x\, \mathcal{L}^\phi$ is invariant under both the gauge and diffeomorphism transformations. We consider only the renormalizable terms for the matter part of the action because they are generic at sub-Planckian energy [37]. For the same reason, for the gravitational part of the Lagrangian density we take R, which is the Lorenz-invariant combination of the metric fields and their derivatives with the lowest energy dimension.

The Hamiltonian for the gravitational part of this action, the Riemann action

$$S^g = \int d^4x\, \frac{\sqrt{-g}}{2}\, R\,, \tag{162}$$

is [32, 39]

$$H^g = \int d^3x\ N^\alpha \mathcal{H}^g_\alpha(\pi^{ij}, \gamma_{ij})\,, \tag{163}$$

where π^{ij} are the operators of the momenta conjugate to the 3-metric γ_{ij},

$$\pi^{ij}(\boldsymbol{x}) = -i\,\frac{\delta}{\delta\gamma_{ij}(\boldsymbol{x})}\,, \tag{164}$$

and $\mathcal{H}^g_\alpha = (\mathcal{H}^{gN}, \gamma_{ij}\mathcal{H}^{gj})$ with [32, 39]

$$\mathcal{H}^{gN} = G_{AB}\,\pi^A\pi^B - \frac{1}{2}\sqrt{\gamma}\ {}^{(3)}\!R\,, \tag{165}$$

$$\mathcal{H}^{gi} = -2\pi^{ij}{}_{|j}\,. \tag{166}$$

In eq. (165) A and B run over all the pairs ij,

$$G_{ij\,kl} \equiv \frac{1}{\sqrt{\gamma}}\,(\gamma_{ik}\gamma_{jl} + \gamma_{il}\gamma_{jk} - \gamma_{ij}\gamma_{kl})\,, \tag{167}$$

${}^{(3)}\!R$ is the Riemann curvature scalar for the 3-metric γ_{ij}, and $\gamma \equiv \det\gamma_{ij}$. For cleaner formulas, we no longer place hats above quantum operators.

Appendix B explicitly demonstrates that the full Hamiltonian that follows from the action (161) is indeed of the form

$$H = \int d^3x\, N^\alpha \mathcal{H}_\alpha\,. \tag{168}$$

Here

$$\mathcal{H}_\alpha = \mathcal{H}^g_\alpha + \mathcal{H}^A_\alpha + \mathcal{H}^\phi_\alpha \tag{169}$$

are local functions of only the fields $f = (\gamma_{ij}, A^a_i, \phi^\alpha)$, of their spatial derivatives, and of the conjugate momenta fields.

Appendix B also shows that

$$e^{-i\int d^3x\, N^i\mathcal{H}_i}\psi(f) = \psi(f - L_{\boldsymbol{N}}f)\,. \tag{170}$$

The corresponding adjoint transformation (144) with $\varepsilon^\mu = (0, \boldsymbol{N})$ transports the field operators $\hat{f}(\boldsymbol{x})$ in space by eqs. (145). Since the wave function by construction satisfies the momentum constraints $\mathcal{H}_i\psi = 0$, eq. (170) shows that the wave function is invariant under the spatial displacement of the fields:

$$\psi(f) = \psi(f - L_{\boldsymbol{N}}f)\,. \tag{171}$$

By the Hamiltonian constraint $\mathcal{H}_N\psi = 0$, the total Hamiltonian (168) for the general $N(\boldsymbol{x})$ also annihilates the wave function. While the corresponding Schrodinger equation then would show no dependence of ψ on time, the Hamiltonian constraint itself specifies ψ evolution [32]. The role of evolution time is now formally[10] taken by the metric scale factor $a(\boldsymbol{x})$ of eq. (160). Indeed, Appendix A of Ref. [32] proves that the operator

$$G_{AB}\,\pi^A\pi^B = -\,G_{AB}\,\frac{\delta^2}{\delta\gamma_A\,\delta\gamma_B} \tag{172}$$

in the gravitational Hamiltonian density (165) is a hyperbolic Laplacian operator with signature $(-+++++)$. The scale factor a corresponds to the "timelike" coordinate of the six coordinates γ_A at every $\boldsymbol{x}$. Namely, from Ref. [32]

$$G_{AB}\,\pi^A\pi^B = \frac{1}{24a}\,\frac{\delta^2}{\delta a^2} - \bar{G}_{\bar{A}\bar{B}}\,\frac{\delta^2}{\delta\zeta_{\bar{A}}\,\delta\zeta_{\bar{B}}} \tag{173}$$

where $\bar{G}_{\bar{A}\bar{B}}$ is positive definite and $\zeta_{\bar{A}}$ are five independent coordinates that parameterize h_{ij} of eq. (160). The Hamiltonian constraint, which is the Wheeler-DeWitt equation with matter

$$\left(\frac{1}{24a}\,\frac{\delta^2}{\delta a^2} - \bar{G}_{\bar{A}\bar{B}}\,\frac{\delta^2}{\delta\zeta_{\bar{A}}\,\delta\zeta_{\bar{B}}} - \frac{1}{2}\sqrt{\gamma}\ {}^{(3)}\!R + \right.$$
$$\left. +\ \mathcal{H}_N^A + \mathcal{H}_N^\phi\right)\psi = 0\,, \tag{174}$$

is thus a hyperbolic equation. As such, it has a unique solution for any initial $\psi(\phi, A_i, h_{ij}, a)$ and $\delta\psi(\phi, A_i, h_{ij}, a)/\delta a$ at some $a = a_0(\boldsymbol{x})$.

We can now identify smooth fitting functions of the generic basic distribution $\nu(q)$ that represent the evolving states of quantum fields in the quasiclassical spacetime of an eternally inflating universe. This is the primary goal of the section next.

VII. EMERGENCE OF THE PHYSICAL WORLD AND INITIAL CONDITIONS

We begin this section with a detailed analysis of how some solutions of the Wheeler-DeWitt equation with matter (174) represent quasiclassical motion of macroscopic degrees of freedom. We then reduce that equation to the Schrodinger equation of quantum evolution on smaller scales. In principle, we could follow either of two formally equivalent approaches. In Appendix C we review a historically more early "Hamiltonian" approach [41]. However, we also see that despite offering helpful intuition, this formalism runs into technical complications that make it less practical for specific applications. In Sec. VII C of the main text we use a more modern "Lagrangian" approach. It allows us to directly incorporate the requirements of the general covariance and offers relatively simple evolution equations. Then matching the full matter and gravitational dynamics to the fitting functions for the generic basic structure becomes straightforward.

A. Emergence of classical trajectories

As discussed the previous section, a suitable evolution parameter for a solution of the Wheeler-DeWitt equation (174) is the conformal scale factor $a(\boldsymbol{x})$ of eq. (160). We write solutions of the Wheeler-DeWitt equation symbolically as $\psi(f)$ where now $f \equiv (\phi, A_i, h_{ij}, a) \equiv (\phi, A_i, \gamma_{ij})$. Given initial ψ and $\delta\psi/\delta a$ for all (ϕ, A_i, h_{ij}) at some $a = a_0$, the Wheeler-DeWitt equation (174) specifies $\psi(f)$ at all other configurations $a(\boldsymbol{x})$.

Let us split the fields $f(\boldsymbol{x})$ into two terms

$$f^a(\boldsymbol{x}) = \bar{f}^a(\boldsymbol{x}) + \tilde{f}^a(\boldsymbol{x}) \tag{175}$$

as follows. Let $\bar{f}$ contain all the modes whose spatial wavelength at some reference moment is larger than a certain borderline value, and let $\tilde{f}$ contain all the remaining modes. We will specify the borderline wavelength later. The variables $\bar{f}$ will represent coherent motion on macroscopic scales. The energy and De-Broyle frequency of this motion will typically exceed the Planck energy by orders of magnitude.

Among various solutions $\psi(\bar{f}, \tilde{f})$ of the Wheeler-DeWitt equation (174) let us consider the ones in which the "macroscopic" degrees of freedom $\bar{f}$ are quasiclassical. We start with the quasiclassical ansatz

$$\psi(\bar{f}, \tilde{f}) = A(\bar{f})\,e^{iS(\bar{f})}\,\tilde{\psi}(\bar{f}, \tilde{f})\,, \tag{176}$$

where $A(\bar{f})$ and $S(\bar{f})$ are real, but $\tilde{\psi}(\bar{f}, \tilde{f})$ can be complex. The wave function (176) is quasiclassical in $\bar{f}$ when the phase rate of change, $\delta S/\delta\bar{f}$, and the prefactor, $A\tilde{\psi}$, vary negligibly in $\bar{f}$ over the increments $\Delta\bar{f}$ that give $\Delta S \sim 1$.

We set the real functions $S(\bar{f})$ and $A(\bar{f})$ in eq. (176) to be respectively the phase and the amplitude of the "background" solution

$$\bar{\psi}(\bar{f}) = A(\bar{f})\,e^{iS(\bar{f})}\,, \tag{177}$$

solving

$$\bar{\mathcal{H}}_\alpha(\bar{f}, \bar{\pi})\,\bar{\psi}(\bar{f}) = 0\,. \tag{178}$$

Here $\bar{\mathcal{H}}_\alpha$ is obtained from $\mathcal{H}_\alpha(\bar{f} + \tilde{f}, \bar{\pi}, \tilde{\pi})$ by dropping the variables $\tilde{f}$ and their conjugate momenta $\tilde{\pi}$. For the

[10] Matching the formal quantum-gravitational evolution in $a(\boldsymbol{x})$ to the observed evolution in quasiclassical physical time is by no means straightforward. This is discussed in Refs. [32, 41, 42] and further in Sec. VII.

leading order of quasiclassical expansion we replace the operators $\bar{\pi} = -i\delta/\delta\bar{f}(\boldsymbol{x})$ in eq. (178) by c-numbers

$$\bar{\pi} = \frac{\delta S}{\delta \bar{f}(x)} \,. \tag{179}$$

This gives the Hamilton-Jacobi equations for $S(\bar{f})$:

$$\bar{\mathcal{H}}_\alpha(\bar{f}, \bar{\pi}) \equiv \bar{\mathcal{H}}_\alpha\left(\bar{f}, \frac{\delta S}{\delta \bar{f}}\right) = 0 \,. \tag{180}$$

Gerlach [42] showed with amazing care how for general relativity without matter the Hamilton-Jacobi equations (180) lead to classical trajectories in the configuration space. The fields evolve along these trajectories by the classical equations of motion for the Hamiltonian

$$\bar{H} = \int d^3x \, N^\alpha \bar{\mathcal{H}}_\alpha(\bar{f}, \bar{\pi}) \,. \tag{181}$$

Later Kim [43] stressed that for consistent quasiclassical description of inflating universe the "background" Hamiltonian components $\bar{\mathcal{H}}_\alpha$ must include not only the long-wavelength metric degrees of freedom $\bar{\gamma}_{ij}$ but also, at least, the long-wavelength part of the field ϕ that drives inflation. Omission of the inflaton field from $\bar{\mathcal{H}}_\alpha$ would lead to *non-oscillating* solution [43] for the background wave function $\bar{\psi}(\bar{f}) \propto e^{iS(\bar{f})}$. Then the quasiclassical requirement of slow variation in $\bar{f}$ of $\delta S/\delta\bar{f}$ and the prefactor in a solution eq. (176) could not be fulfilled [43].

It is straightforward to generalize Gerlach's arguments [42] to $\bar{f}$ and $\bar{\mathcal{H}}_\alpha$ that include both the metric and matter fields. The observed classical trajectory of the system (178) is the set of the fields $\bar{f}_{\rm cl}$ that are points of constructive interference of solutions of the quantum constraint equations (178). Consider the solutions of eqs. (178) that are waves of the form

$$\psi_{\bar{\pi}}(\bar{f}_0 + d\bar{f}) \propto e^{i\bar{\pi}\cdot d\bar{f}} \,, \tag{182}$$

where

$$\bar{\pi} \cdot d\bar{f} \equiv \int d^3x \sum_\alpha \bar{\pi}_\alpha(\boldsymbol{x}) \, d\bar{f}^\alpha(\boldsymbol{x}) \,. \tag{183}$$

These waves with wavenumbers $\bar{\pi}$ in a narrow range $(\bar{\pi}_{\rm cl} - \delta\bar{\pi}, \bar{\pi}_{\rm cl} + \delta\bar{\pi})$, centered at some $\bar{\pi}_{\rm cl}$, interfere constructively [42] along an interval $d\bar{f}_{\rm cl}$ of a trajectory in the configuration space when

$$\delta\bar{\pi} \cdot d\bar{f}_{\rm cl} = 0 \,. \tag{184}$$

This condition is illustrated by Fig. 3.

In the quasiclassical limit, the phase $\bar{\pi} \cdot d\bar{f}$ of every solution (182) obeys the Hamilton-Jacobi equations (180). Following Ref. [42], we use the Lagrange multiplies for finding the extremum (184) under the constraints (180) for every α and $\boldsymbol{x}$. Thus we require for all infinitesimal $\delta\bar{\pi}$ the following variation to vanish:

$$\delta\bar{\pi} \cdot d\bar{f}_{\rm cl} - \int d^3x \, dt \, N^\alpha \delta \bar{\mathcal{H}}_\alpha(\bar{f}_{\rm cl}, \bar{\pi}) = \\ = dt \int d^3x \; \delta\bar{\pi} \left[\dot{\bar{f}}_{\rm cl} - N^\alpha \frac{\partial \bar{\mathcal{H}}_\alpha(\bar{f}_{\rm cl}, \bar{\pi}_{\rm cl})}{\partial \bar{\pi}_{\rm cl}} \right] = 0 \,. \tag{185}$$

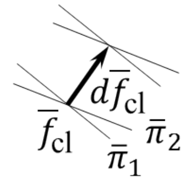

FIG. 3: An interval $[\bar{f}_{\rm cl}, \bar{f}_{\rm cl} + d\bar{f}_{\rm cl}]$ of a classical trajectory that appears from constructive interference of waves $\psi_{\bar{\pi}} \propto e^{i\bar{\pi}\cdot d\bar{f}}$ with various momenta $\bar{\pi}_1$, $\bar{\pi}_2, \ldots$ in a range $\bar{\pi}_{\rm cl} \pm \delta\bar{\pi}$. The thin lines represent the constant-phase surfaces of the waves, e.g., their crests. The waves add up along the classical trajectory, depicted by the thick arrow.

Here $dt\, N^\alpha(\boldsymbol{x})$ are the Lagrange multiplies, $\dot{\bar{f}}_{\rm cl} \equiv d\bar{f}_{\rm cl}/dt$, and for a given $d\bar{f}_{\rm cl}$ the value of dt is determined by our choice of $N^\alpha(\boldsymbol{x})$. Condition (185) yields half of the Hamilton equations of classical motion:

$$\dot{\bar{f}}_{\rm cl}(\boldsymbol{x}) = \frac{\delta \bar{H}(\bar{f}_{\rm cl}, \bar{\pi}_{\rm cl}, N^\alpha)}{\delta \bar{\pi}_{\rm cl}(\boldsymbol{x})} \,, \tag{186}$$

with $\bar{H}$ of eq. (181). The other half of the classical Hamilton equations,

$$\dot{\bar{\pi}}_{\rm cl}(\boldsymbol{x}) = - \frac{\delta \bar{H}(\bar{f}_{\rm cl}, \bar{\pi}_{\rm cl}, N^\alpha)}{\delta \bar{f}_{\rm cl}(\boldsymbol{x})} \tag{187}$$

then also follows as shown in Ref. [42]. The classical equations of motion (186–187) should be complemented by the constraints (180):

$$\frac{\delta \bar{H}(\bar{f}_{\rm cl}, \bar{\pi}_{\rm cl}, N^\alpha)}{\delta N^\alpha(\boldsymbol{x})} = \bar{\mathcal{H}}_\alpha(\bar{f}_{\rm cl}, \bar{\pi}_{\rm cl}) = 0 \,. \tag{188}$$

The lapse and shift functions $N^\alpha(x)$ can be chosen arbitrarily. Their choice, however, affects $\bar{f}_{\rm cl}(x)$ and $\bar{\pi}_{\rm cl}(x)$ that result from the evolution by eqs. (186–187). The configurations $\bar{f}_{\rm cl}(t, \boldsymbol{x})$ for any $N^\alpha(x)$ are points of constructive interference of the waves (182). The set of all such configurations can be parameterized by the Tomonaga "many-fingered time" $\sigma(\boldsymbol{x})$ [44] as $\{\bar{f}_{\rm cl}(\sigma(\boldsymbol{x}), \boldsymbol{x})\}$ [42]. This set may be interpreted physically as the configurations of the classically evolving fields on all the various spatial slices $(t = \sigma(\boldsymbol{x}), \boldsymbol{x})$ of the physical spacetime.

B. Hamiltonian description of quantum components

We return to the full quantum system $\psi(f)$, satisfying the constraints $\hat{\mathcal{H}}_\alpha \psi(f) = 0$. In Appendix C we utilize the approach of Lapchinsky and Rubakov [41] to obtain a Schrodinger-like evolution equation for the small-scale "quantum" degrees of freedom $\tilde{f}$:

$$i\partial_t \tilde{\psi} = \tilde{H} \tilde{\psi} \tag{189}$$

with $\tilde{H} = (\int d^3x\, N^\alpha \mathcal{H}_\alpha) - \bar{H}$. However, Appendix C also shows that while eq. (189) *looks* like a time-dependent Schrodinger equation for a wave function $\tilde{\psi}(\tilde{f}, t)$, it is rather a formal result that is not a stable Schrodinger equation yet. To be used as such, it requires removing spurious negative kinetic energy terms and resolving some other issues, described in Appendix C. This substantially complicates the formalism that is based on the direct quasiclassical expansion of the Wheeler-DeWitt equation.

C. Lagrangian description of quantum components

For applications and derivation of a stable Schrodinger equation of quantum field theory it is more convenient to use the Lagrangian formulation of quantum-gravitational dynamics on quasiclassical background (e.g., [45–47]). We already encountered the Lagrangian formulation of quantum dynamics through the Feynman path integral in the earlier sections. In particular, in Sec. VI we used the action (161) to find Hamiltonians for the systems with gauge and diffeomorphism symmetries.

Classical trajectories $\bar{f}_{\rm cl}$ (186–187) of the "macroscopic" variables $\bar{f}$ extremize the action $\int dt\, \bar{L}$ of the "background" degrees of freedom:

$$\left(\partial_t \frac{\delta \bar{L}}{\delta \dot{\bar{f}}(\boldsymbol{x})} - \frac{\delta \bar{L}}{\delta \bar{f}(\boldsymbol{x})}\right)\bigg|_{\bar{f}_{\rm cl}} = 0\,. \tag{190}$$

Here $\bar{L} \equiv L(\dot{\bar{f}}, \bar{f}, N^\alpha)$ is related to $\bar{H}$ of the previous subsection by the Legendre transformation. The classical background solutions also obey the primary constraints

$$\frac{\delta}{\delta N^\alpha(\boldsymbol{x})}\, \bar{L}(\dot{\bar{f}}_{\rm cl}, \bar{f}_{\rm cl}, N^\alpha) = 0\,. \tag{191}$$

(They correspond to the constraints $\bar{\pi}_{N^\alpha}\bar{\psi} = 0$ of the Hamiltonian formulation.) By eq. (6), N^α are in one-to-one relation with the four components $g^{0\mu}$ of the inverse metric tensor. Hence for the Lagrangian of general relativity with matter the constraints (191) give four of the ten independent components of the Einstein equations for the background fields $\bar{f}_{\rm cl} = (\bar{\phi}, \bar{\gamma}_{ij}, \dots)_{\rm cl}$:

$$\bar{G}_{0\mu} - \bar{T}_{0\mu} = 0\,, \tag{192}$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R$ and $R_{\mu\nu}$ is the Riemann tensor.

We now return to the full system, including the short-scale quantum degrees of freedom $\tilde{f}$. We start from a local Lagrangian

$$L = \int d^3x\ \mathcal{L}(\dot{f}(\boldsymbol{x}), f(\boldsymbol{x}), N^\alpha) \tag{193}$$

that yields a generally covariant action $S = \int dt\, L$. Let us in the path integral use only the field configurations that obey the primary constraints

$$\frac{\delta}{\delta N^\alpha(\boldsymbol{x})}\, L(\dot{f}, f, N^\alpha) = 0\,. \tag{194}$$

If we set $\bar{f}$ to the classical trajectories $\bar{f}_{\rm cl}$ then, by eqs. (191) and (194), we have for

$$\tilde{L}(\dot{f}, f, N^\alpha) \equiv L - \bar{L} \tag{195}$$

a similar constraint:

$$\frac{\delta}{\delta N^\alpha(\boldsymbol{x})}\, \tilde{L}(\dot{f}, f, N^\alpha)|_{f = \bar{f}_{\rm cl} + \tilde{f}} = 0\,. \tag{196}$$

Consider a derived Lagrangian

$$\tilde{L}(\dot{\tilde{f}}, \tilde{f}, t) \equiv \tilde{L}(\dot{f}, f, N^\alpha(t))\Big|_{f = \bar{f}_{\rm cl}(t) + \tilde{f}}\ , \tag{197}$$

where $\bar{f}_{\rm cl}(t)$ is regarded as an external function rather than a dynamical degree of freedom and where the dynamical fields $\tilde{f}$ are constrained by eq. (196). This Lagrangian by itself specifies viable evolution in the space of fitting functions (54) for the generic basic structure.

As discussed in the earlier Sec. VI B, we wish to match the coordinates of the fitting functions (54) to the 3-diffeomorphism orbits of the physical fields. To this end we take a 4-diffeomorphism invariant action $S = \int dt\, L$ and construct with eqs. (195–197) the Lagrangian $\tilde{L}$ for the evolution of $\tilde{\psi}(\tilde{f}, t)$. The background field $\bar{f}_{\rm cl}(x)$ in these equations is any function that obeys the classical equations (190–191) for the same action S.

A change of spatial coordinates corresponds to a joint 3-diffeomorphism transformation of both $\tilde{f}$ and $\bar{f}_{\rm cl}$ with a common displacement $\varepsilon^\mu(x)$. Since the diffeomorphism transformation (145) is linear and homogeneous in the transformed field, the joint transformation of $\tilde{f}$ and $\bar{f}_{\rm cl}$ is also a diffeomorphism transformation of the full field $f = \bar{f}_{\rm cl} + \tilde{f}$. After such a transformation the new Lagrangian $\tilde{L}'$ for quantum evolution is given by the same construction (195–197) but with the transformed background $\bar{f}'_{\rm cl}$. In general, $\tilde{L}'$ and $\tilde{L}$ will be different functions.

We now map the basic coordinates of the smooth fitting functions (54) to the 3-diffeomorphism orbits of the joint transformation of $\tilde{f}$ and $\bar{f}_{\rm cl}$. For various choices of spatial coordinates we encounter generally different but equivalent Lagrangians $\tilde{L}$ and wave functions $\tilde{\psi}(\tilde{f}, t)$ for the quantum degrees of freedom $\tilde{f}$. The wave functions in the various coordinate frames are all represented by the same underlying basic structure. These representations are equivalent but their form varies with the coordinate frame.

Gauge transformations of gauge connection fields $\boldsymbol{A}$, eq. (120c) or (A15), are not homogeneous in the transformed field. For this reason their separation into classical and quantum parts is less straightforward. We can avoid the related complications by simply delegating all

the modes of the gauge fields to the quantum degrees of freedom. Of course, this does not prevent some modes of $\boldsymbol{A}$ from becoming de facto classical by constructive interference of quasiclassical waves, Sec. VII A. Charged matter fields ϕ^α transform homogeneously under gauge transformations, eqs. (120a–120b) or (A5). We therefore can straightforwardly decompose them into classical and quantum parts, $\phi^\alpha = \bar\phi^\alpha_{\rm cl} + \tilde\phi^\alpha$, and construct a quantum physical system with the Lagrangian $\tilde L$ as described above.[11]

Consider a diffeomorphism-invariant wave function $\psi(\bar f, \tilde f)$ that solves the Wheeler-DeWitt equation (174). Let the solution be quasiclassical in the variables $\bar f$ in a band of spatial hypersurfaces around certain time t. For example, we can think of the wave function that describes eternal inflation. This wave function is typically a superposition of numerous decohered Everett's branches with various classically evolving long-wavelength modes, $\bar f_{\rm cl}(t)$. The evolution of $\tilde\psi(\tilde f, t\,|\,\bar f_{\rm cl})$ from eq. (176) in any individual Everett's branch is the same whether we match the modes $\bar f$ to the coordinates of the underlying basic structure or treat them as external classical fields $\bar f_{\rm cl}$. Thus the observed local world can be represented not only by the above $\psi(\bar f, \tilde f)$ but also by a pair $(\bar f_{\rm cl}, \tilde\psi)$, where $\bar f_{\rm cl}(x)$ evolves by the classical Euler-Lagrange equations (190) for the Lagrangian $\bar L$ and $\tilde\psi(\tilde f(\boldsymbol{x}), t)$ evolves by the path-integral transformation with the Lagrangian $\tilde L$ (195–197), obtained from the original diffeomorphism-invariant local action $S = \int dt\, L$.

Likewise, the same physical world is simultaneously represented by different pairs $(\bar f_{\rm cl}, \tilde\psi)$ that are indistinguishable to a local observer. Indeed, the typical physical observer is described by a decohered Everett's branch with a definite $\bar f_{\rm cl}(t)$. This observer cannot distinguish representation of some of the field modes $\bar f_{\boldsymbol{k}}$ by c-number function $\bar f_{\rm cl}(t)$ from representation of the same modes by quantum variables $\tilde f$ that evolve quasiclassically. The physical world is composed of all these representations, given by different fitting functions for $\nu(q)$, that satisfy

$$\langle i|i\rangle \gg \delta\chi^2_{\rm min} \tag{198}$$

for their objective existence [cf. text around eq. (51)]. Thus the observed world is the *ensemble* of patterns within the generic basic structure $\nu(q)$ that provide observationally indistinguishable representations of a physical state.

The four constraints (196) allow us to express four of the six components of $\tilde\gamma_{ij}$, including the scale factor $\tilde a$, as functions of the remaining dynamical fields and the arbitrary lapse and shift $N^\alpha(x)$. As a result, in the Lagrangian formulation we do not encounter the spurious "negative kinetic energy" term $\delta^2/\delta\tilde a^2$, which complicated the Hamiltonian formulation as reviewed in Appendix C. In the Lagrangian approach the only dynamical components of $\tilde\gamma_{ij}$ are manifestly the two physical polarizations of gravitons with positive kinetic energy.

The described Lagrangian formulation with constraints was applied to calculate the joint evolution of background classical and small-scale quantum fields during inflation in Refs. [45, 46], working in the linear order in $\tilde f$. Ref. [47] extended the approach of [45] to higher, non-linear orders. In the companion paper [33] we employ the presented formulation to analyze black hole evaporation.

D. Gravitationally collapsing regions

During quasiclassical general-relativistic evolution some regions of the universe gravitationally collapse. The collapse is unstoppable under conditions [48, 49] that are commonly expected in the central parts of massive stars, galactic cores and other cosmological overdense regions. Physical laws should fully describe if not necessarily the full interior of such regions then, at least, interaction of their boundary with the remaining universe throughout its entire evolution. This applies to both the classical and quantum components of the pairs $(\bar f_{\rm cl}, \tilde\psi)$.

In further Sec. VIII we argue that emergent systems that represent viable physical worlds should inherently possess dynamical *local* symmetries. In companion Ref. [33] we also see that a diffeomorphism-symmetric action of general relativity is sufficient for describing the classical-quantum evolution $(\bar f_{\rm cl}, \tilde\psi)$ outside a black hole throughout its entire evolution and ultimate complete evaporation. Specifically, the pair $(\bar f_{\rm cl}, \tilde\psi)$ can evolve by a generally covariant quantum field theory both outside and inside the black hole event horizon until the geometrical curvature invariants (R, $R_{\mu\nu\lambda\chi}R^{\mu\nu\lambda\chi}$, etc.) approach the Planck scale (m_P^2, m_P^4, etc., matching the dimension of the invariant). While the black hole evaporates by emitting the Hawking radiation [50], its event horizon shrinks. The curvature invariants outside the horizon remain sub-Planckian until the final stage of the evaporation, when the black hole mass diminishes to the order of m_P. And even then and afterwards there is dynamically unambiguous and physically acceptable continuation of the evolution of $(\bar f_{\rm cl}, \tilde\psi)$ at the distances $r \gtrsim m_P^{-1}$ away from the vanishing black hole. According to Secs. II and VIII of this paper, as long as such an unambiguous path of evolution exists, it is objectively realized in nature.

Although here and in Ref. [33] we consider only bosonic

[11] Evolution of the classical component $\bar f_{\rm cl}$, by its construction in eqs. (190–191), is not influenced by the quantum degrees of freedom. Therefore, during evolution $\bar f_{\rm cl}$ may significantly diverge from the associated physical classical objects, represented by both $\bar f_{\rm cl}$ and the quasiclassical modes of $\tilde f$. In particular, the described formalism, where the gauge fields are fully quantum and do not affect the classical charges, would be inconvenient for studying the motion of charged classical objects. On the other hand, this formalism is useful when classical evolution is predominantly controlled by gravitational interaction, such as during cosmological inflation or black hole evaporation.

fields, we expect that fermions do not affect our conclusions for the following reasons. The geometry of spacetime at sub-Planckian curvature may be consistently described by the quasiclassical metric. Of course, our world might be more intricate and non-metric gravitational degrees of freedom might also become relevant at sub-Planckian energy. But if it is not the case, i.e., if general relativity is suitable to all energies well below m_P then fermions contribute [51] to the Hawking radiation similarly to bosons. Any non-metric gravitational, e.g. supergravitational, degrees of freedom are then excited only at the very final stage of black hole evaporation. Except for the added possibility of a black hole remnant with a mass of the order of m_P, this does not influence the results of either this paper or the companion Ref. [33].

We thus proceed under the plausible assumption that unknown physics does not invalidate the field-theoretical and general-relativistic descriptions of our world at any energy density well below the Planck scale. As we will see soon, then mapping the physical quantum fields to the underlying basic structure becomes particularly simple around the Planck energy.

E. Emergence of inflating worlds

Our goal is to identify in the generic basic structure $\nu(q)$ of Secs. III and IV C a class of physically-equivalent pairs $(\bar{f}_{\rm cl}, \tilde{\psi})$ that can represent the observed world. Abundant empirical evidence suggests that the visible universe has evolved from inflationary past. Indeed, not only has the inflationary paradigm naturally justified [23] the observed spatially-flat, nearly homogeneous, low-entropy cosmological initial conditions. But inflation has also been tremendously successful in having *predicted* a variety of properties of the cosmological structure that were unknown at the time. This includes "adiabaticity" of the primordial cosmological perturbations, their approximate Gaussianity, and the small and almost scale-independent tilt of their power spectral index $n_s - 1$ [52–54].

Accordingly, we look for pairs $(\bar{f}_{\rm cl}, \tilde{\psi})$ that correspond to inflationary or post-inflationary configurations of physical fields. Let in these pairs $\bar{f}_{\rm cl}(t, \boldsymbol{x})$ describe a classical background that is smooth on the length scales comparable to or smaller than the Hubble radius. Let $\tilde{\psi}[\tilde{f}(\boldsymbol{x}), t]$ be the wave function of the inhomogeneities on these, horizon or subhorizon, scales. As we see next, in inflating background metric the quantum fields $\tilde{f}(\boldsymbol{x})$ indeed map naturally to the coordinates of the generic basic structure. A suggestive hint for this is that both the fitting function for the generic distribution of Sec. IV C and the primordial distribution of the inhomogeneity modes, as far as the current cosmological observations show, have the same Gaussian form. We nonetheless should exercise care in linking the two distributions. In particular, the generated by inflation initial conditions for even negligible non-gravitational interaction of the fields are expected to be Gaussian only approximately [47].

It might be tempting to identify the initial wave function of the modes with $\psi_0(q)$ of eq. (82) that emerges in the generic basic structure $\nu(q)$. Such an identification would naturally answer two important questions that have not been explained in the earlier literature. The first of them is even referred by some authors as suggesting problems [25, 26] with the inflationary paradigm.

The first question is why all the field modes evolve from highly special low-entropy initial conditions, necessary for inflation. Any light field whose quantized modes are characterized by an occupation number $n(k)$ increases energy density, ε, of radiation by

$$\Delta\varepsilon_{\rm rad} = \int d\!\!{}^-{}^3k\, k\, n(k) \sim \bar{n}\, m_P^4\,. \tag{199}$$

Here $\bar{n}$ is the average energy-weighted occupation number of the modes. Since for the radiation pressure $p_{\rm rad} = \varepsilon_{\rm rad}/3$, the condition for the total pressure during inflation $p \leqslant -\varepsilon/3$ [55] holds only if $\bar{n} \ll 1$, i.e., only if the high-frequency modes emerge from the Planck scale essentially in the vacuum state.

The second question is why inflation, which generically continues forever [24], does not "run out" of the degrees of freedom while the space and dynamical fields in it expand to arbitrarily large extent.

Regarding the first question, Sec. IV C demonstrated that if the wave function fundamentally is the fitting function of the generic structure then in certain basic coordinates q it will indeed be of the specific Gaussian form. This agrees with the inflationary requirement that the initial state of high-energy modes is highly specific. We, nevertheless, remember that in the presence of interaction, even gravitational one, this specific ground state usually does not have strictly Gaussian wave function [47]. This discrepancy can be substantial when the interaction becomes significant, as expected even for gravity around the Planck scale. We will resolve this issue by the end of this subsection.

The second question—the origin of the new microscopic degrees of freedom during perpetual inflation—is naturally answered as follows. In the described construction the wave function of the new modes, emerging during inflation from ultra-small scales, stems from fitting the basic structure by a function of basic variables q^n. The physical universe is then the ensemble of all the different choices for q^n, properly normalized but arbitrarily directed. For a finite number of independent basic coordinates the new modes in eternal inflation will eventually be represented by coordinates q^n that are already involved in representing other modes on larger scales, including superhorizon scales. Thus inflation can still continue eternally even for a finite number of the basic objects and their properties. In its course, however, the

basic-level information[12] then have to be *recycled* from superhorizon to the newly emerging microscopic scales.

To find fitting functions (54) that are suitable candidates for the observed world, we need to resolve several, ultimately interrelated, issues. First, as already noted, if we attempted to associate the initial conditions of the field modes in an inflating universe with the generic Gaussian wave function $\psi_0(q)$, eq. (82), then for a realistic interacting theory we would generally find that this ψ_0 is not the ground state of the Hamiltonian. Second, energy of the modes (with respect to comoving coordinates) redshifts during the cosmological expansion. Then the wave function of the modes changes its shape. In an interacting theory the form of the wave function changes even on deeply subhorizon scales. Hence we should ask at which energy, if any, the modes' wave function matches $\psi_0(q)$. Third, in patches where inflation has ended, overdense regions can and in our universe do gravitationally collapse. In the collapsing regions the wave function of progressively more energetic modes departs from the vacuum. Then how can we identically set the modes' wave function at some energy to a fixed function?

As already stated, we suppose that the physical world, including the exterior of collapsing regions, can be described at any energy density below the Planck scale, $\varepsilon \ll m_P^4$, by a field theory. In this section we will refer to the energy density ε in the rest frame, where the total momentum density is zero. More rigorously, we could quantify the field configurations by the curvature invariants as outlined in Sec. VII D.

Ref. [33] shows in detail that the classical-quantum evolution by the general-relativistic action (161) continues unambiguously outside the (stretched) horizon of a black hole with mass $M \gtrsim m_P$ through its complete evaporation. On the other hand, in general relativity field *excitations* with $\varepsilon \sim m_P^4$ over the ground state inevitably collapse to black holes. At the Planckian energy density they cannot evolve further by the considered field theory. Thus the physical wave function of the short-scale modes whose even lowest excitations give $\varepsilon \sim m_P^4$ should always describe the only possible physical state of these modes—the ground state. This unique physical wave function can then be matched to the generic basic fitting function (80) with a fixed rule, considered below. The referred modes with a unique (for a given Hamiltonian) wave function have frequency of the order of m_P.

A mode frequency ω that starts at $\omega \sim m_P$ redshifts during cosmological expansion to sub-Planckian values. For the nearly adiabatic evolution of the high-frequency modes that are well inside the Hubble horizon, their occupation number remains approximately constant. We map new coordinates q^n to the amplitudes of the new physical modes so that the modes emerge at the Planck frequency in the ground state. Then the strict necessary condition for inflation $\bar{n} \ll 1$, where $\bar{n}$ is the average occupation number (199), is satisfied automatically. Opposite to cosmological expansion, the contraction of overdense regions then also proceeds by well defined, continuous, and as argued in Sec. VIII unique dynamics. The resulting quantum evolution during gravitational collapse and complete evaporation of the formed black holes is described in the companion paper [33].

The ground state of the modes with frequency near the Planck scale may not in a given interacting theory have the Gaussian wave function $\psi_0(q)$. Nevertheless, we are free to assign the initial wave function of the modes that emerge at $\omega \sim m_P$ to any of the equivalent representations (54), e.g., to

$$\psi_0'(q) = \hat{U}_0\, \psi_0(q)\,, \tag{200}$$

where $\hat{U}_0$ is an arbitrary non-degenerate operator. In a consistent emergent theory, the operator $\hat{U}_0$ yields ψ_0' that is the ground state of the Planck-frequency modes for the theory Hamiltonian. We can maintain the canonical form of the Hermitian product (79) by choosing a unitary operator $\hat{U}_0$ that yields the desired ψ_0'. Another option to arrive at the necessary initial ψ_0' is to apply a non-degenerate transformation to ρ of eq. (39) and then again to construct the corresponding canonical wave function with eq. (53).

Suppose that there is no antropically acceptable Hamiltonian with the ground state of the Planck-frequency modes in the Gaussian form ψ_0. If, in addition, there exists a unique antropically suitable Hamiltonian with a ground state ψ_0' of the form (200) then, trivially, it becomes the unchallenged candidate for representing our physical world. But if there exist antropically acceptable Hamiltonians of both types then the one associated with ψ_0' would not as likely represent our world as would the Hamiltonian with the ground state ψ_0. We postpone this topic until a future paper.

Rather than arriving at the ground state ψ_0' (200) by transforming the generic Gaussian ψ_0, we could pick a special, non-typical subset of the basic fundamental objects that would directly yield the desired $\psi_0'(q)$. We pointed this out in footnote 9 of Sec. V. Yet while this is true, there appears no justification to limit the physical world to such exclusive subsets. Indeed, in parallel there exist many more other generic sets of the objects that contain the same emergent physical world through the representation (200).

F. Local mixed states

Finally, and importantly for the resolution [33] of the black hole paradoxes [29, 31], we observe that the emergent quantum-field world is necessarily described by a density matrix, rather than by a pure wave function. Indeed, the emergent physical world is the ensemble of all

[12] "Recycling" refers to the basic information in the discrete distribution rather than the quantum information [1] in the physical wave function. The smooth ground-state wave function of the emergent modes carries no information about the larger scales.

the field states that match our current local environment. The degrees of freedom beyond today's Hubble horizon should be traced over, yielding the locally-relevant density matrix. Trace should also be taken over the degrees of freedom under the event horizons of the black holes in the visible universe.

We saw in Sec. III that a wave function $\psi[f(\boldsymbol{x})]$ that emerges from a fitting function for a basic distribution $\nu(q)$ exists not merely as a mathematical abstraction but as an objective entity if and only if condition (198) holds. For a pure wave function the criterion (198) strongly depends on how we extend the equal-time spatial hypersurfaces to superhorizon scales. Yet this criterion is unambiguous for the density matrix of a subhorizon region and observers in it who can maintain causal connection with each other. We describe this quantitatively in further Sec. IX, where we quantify the objective probability of various macroscopic outcomes of a quantum process.

VIII. WHY A SPECIFIC HAMILTONIAN?

We now address the important question of why the particle and other experiments, observations in astrophysics and cosmology, as well as our daily experience consistently show that the physical evolution is governed by a specific Hamiltonian whose couplings do not vary with spacetime location and with the type of an experiment that measures them. We observed in Sec. II that the standard quantum mechanics permits evolution $\psi \to \psi' = \hat{U}\psi$ of any quantum state ψ by an operator $\hat{U} = \exp(-i\hat{H}'\Delta t)$ with an arbitrary $\hat{H}'$. It followed that ψ' "exists" as much as the original wave function ψ does. The Heisenberg time-dependent operators that evolve by $\hat{H}'$ likewise exist. Then, if evolution by the arbitrary $\hat{H}'$ takes place objectively, why do we live through a highly specific version of evolution, with the symmetries and the numerical parameters of the Hamiltonian being constant?

To answer this question, let us remember that in a gauge- and diffeomorphism-invariant theory the arguments of the wave function are the whole symmetry-group orbits [32] rather than individual field configurations. Hence in the corresponding emergent theory we match the points with specific values of the coordinates q of the underlying structure to the entire symmetry orbits of the particle fields. A Hamiltonian that does not preserve the constancy of the wave function on these orbits cannot unambiguously continue the wave function to a new time. It may then be impossible to modify the Hamiltonian of the considered emergent system infinitesimally while preserving the system independent degrees of freedom, specifically their number. Let us call a quantum system *isolated* if it is impossible to change its Hamiltonian infinitesimally within the dimensionality of the system configuration space.

We may, of course, consider other matches where different field configurations (e.g., $f_1 \neq f_2$) that can be transformed into each other by a gauge transformation map to different values (e.g., $q_1 \neq q_2$) of the coordinates q of the underlying basic structure. Then we can evolve the resulting wave function $\psi(f)$ with Hamiltonians that violate the symmetry. However, the resulting new quantum system has no specific direction for its dynamical evolution. On its typical evolution path the physical laws change inherently unpredictably. Its internal subsystems cannot evolve biologically because their past experience does not help them to cope with future challenges. Thus this emergent system, different from the previous symmetric system because of its different degrees of freedom and different dimensionality of the configuration space, does not represent an antropically acceptable physical world.

The arguments above do not apply directly to symmetries that mix fields on separate spatial slices of different time t. In particular, we cannot require the wave function to be unchanged under the 4-diffeomorphism symmetry transformations (145–147) with $\varepsilon^0 \neq 0$. This symmetry is, nevertheless, physically essential. It specifies the evolution through the Wheeler-DeWitt equation (174) and it leads to the observed local Lorentz symmetry. The latter ensures that a localized excitation over the vacuum in one spacetime coordinate frame appears in another frame as a similarly localized excitation with equivalent macroscopic content. In both frames the excitation evolves by the same laws. Without the Lorentz symmetry of the dynamical laws and of the vacuum state, a localized system, including an intelligent observer, would generally loose its identity as soon as it starts moving relative to some global frame. This would highly complicate macroscopic causal relations in interdependent evolution of macroscopic objects, including the development of intelligent observers.

We may discuss at least two alternatives for preserving the local Lorentz symmetry. One is that the directions of evolution that break this symmetry are excluded by anthropic considerations because of the reasons described in the previous paragraph. In this case, alternative evolutionary branches spring off at every moment but no observer in these branches survives for a noticeable duration of time.

Another, more appealing, possibility is protection of the 4-diffeomorphism symmetry by a deeper symmetry. Consider a new symmetry such that any infinitesimal 4-diffeomorphism transformation can be presented as a sequence of the new symmetry transformations that involve only fields on the current spatial hypersurface. The emergent physical systems with this symmetry necessarily evolve by a unique Hamiltonian.

An example is the local supersymmetry. The generators of supersymmetry, Q^A_α, fully determine the Hamiltonian through the anticommutation relation

$$\{Q^A_\alpha, Q^{B\dagger}_\beta\} = 2\delta^{AB}\sigma^\mu_{\alpha\beta}P_\mu\,. \tag{201}$$

Here $A, B = 1, \ldots, N_{\rm SUSY}$; $\sigma^\mu = (-1, \sigma^i)$, where σ^i are the Pauli matrices; and P_μ is the 4-momentum operator.

Trace of eq. (201) yields the Hamiltonian

$$H \equiv P^0 = \frac{1}{4N_{\text{SUSY}}} \sum_{A,\alpha} \{Q^A_\alpha, Q^{A\dagger}_\alpha\}\,. \tag{202}$$

Fermionic fields and quantum states with local supersymmetry exist naturally and compactly in representations of the *same* generic distribution of properties of any objects with real continuous fitting functions of the considered form (54). The description of this result is deferred to a forthcoming paper.

We can locally super-transform the fields or states of an emergent system along any fermionic parameter field $\xi^\alpha(\boldsymbol{x})$. Since ξ^α is not a dynamical field but an arbitrary transformation parameter, a locally supersymmetric wave function cannot depend on it. Similarly to the gauge symmetry (Secs. V B and V C) or diffeomorphism symmetry (Sec. VI C), the requirement of wave function independence from ξ^α after an infinitesimal step of evolution leads to the usual secondary constraint: local supersymmetric transformation, generated by $\int d^3x\, [\xi^\alpha j^0_\alpha + \text{(its conjugate)}]$, should not change the wave function. [Cf. eq. (133) for gauge symmetry or eqs. (157–158) for diffeomorphism symmetry.] Hence the global supersymmetry generator $Q_\alpha = \int d^3x\, j^0_\alpha$ and its conjugate should also annihilate a wave function of a locally supersymmetric system:[13]

$$Q\psi = Q^\dagger\psi = 0\,. \tag{203}$$

These conditions entail by eq. (202) the Hamiltonian constraint $H\psi = 0$, providing unambiguous dynamical evolution of the given system.

Thus, unlike our earlier situation for diffeomorphism symmetry (Sec. VI C), the Hamiltonian constraint is no longer an independent requirement for evolution of the wave function. Rather it is a property of the emergent wave function at any fixed time. We can understand this intuitively by interpreting eq. (202) as suggested in Sec. 6 of Ref. [56]. Namely, each of the infinitesimal supersymmetry transformations by Q or $Q^\dagger$ constitutes "half-step" of a temporal displacement by the Hamiltonian H. Therefore, constancy of the wave function at the half-step ensures its constancy at the full step of temporal displacement.

Of course, there exist various supersymmetric theories with different Hamiltonians. However, for each of them the Hamiltonian of the corresponding emergent system is uniquely encoded in the map of the generic basic structure to the supersymmetry orbits of a given theory. We therefore may expect coexistence of different emergent systems. Yet every of those systems evolves by its own Hamiltonian, unchanged for the states of that system during their evolution.

Similar reasoning applies to the observed constancy of the physical couplings in spacetime [22]. The gauge and diffeomorphism symmetries are compatible with spacetime variation of the couplings provided that under the diffeomorphism transformations the couplings change as scalar fields. Yet the experiments indicate that the coupling are constant. The discussion above suggests, likewise, two different possible explanations.

One is that the observed couplings are tuned to the special values such that even their slight deviation *quickly* destroys the macroscopic structure of our world. It is insufficient that anthropically acceptable environment would not arise through physical evolution at different values of the couplings, e.g., due to the absence of the resonant triple-α process in the stellar nucleosynthesis, or non-occurrence of similar fortunate events in our past. Even a recent detectable spacetime variation of the couplings should make the world uninhabitable so quickly that we have no time to realize our living through these dead-end directions of physical evolution.

Another, more robust, option is again the protection of the constancy of couplings by a symmetry that connects their values at separate spacetime points by a sequence of equal-time symmetry transformations. An example of such a symmetry is again the supersymmetry, of course, broken dynamically at the currently accessible energy scales.

IX. PROBABILITY AND THE BORN RULE

A. Conditions for the Born rule

Let a wave function ψ split during its physical evolution into a sum of orthogonal terms:

$$\psi = \sum_i \psi_i\,, \tag{204}$$

$$\langle\psi_i|\psi_j\rangle = \delta_{ij} w_i\,. \tag{205}$$

The Everett branches ψ_i after the split for a physical system with a large number of coupled environmental degrees of freedom rapidly decohere.

In Sec. III we considered wave functions $\psi(Q,t)$ that fundamentally are the coefficients in linear combinations of smooth basis function (54) that fitted a generic finite structure. Then we found in the same Sec. III that an Everett's branch ψ_i exists as an objective entity if and only if its weight $w_i \equiv \langle\psi_i|\psi_i\rangle$ exceeds a certain positive threshold $\delta\chi^2_{\min}$.

Suppose that in the accessible to us patch of the universe someone performs a multiple-outcome quantum

[13] For preventing possible confusion, let us note that eq. (203) does not imply vanishing the supersymmetric charge of every particle. Analogously, the similar constraint of a gauge theory does not imply the absence of charged particles. Likewise, the Hamiltonian and momentum constraints $P_\mu\psi = 0$ of canonical quantum gravity do not require the energy and momentum of every particle to be zero. These equations only state that the *simultaneous* transformation of the matter, gauge, and metric fields does not change the overall wave function.

experiment, e.g., the Stern-Gerlach, or double-slit, or Schrodinger cat experiment. The experimental object, apparatus, local environment, experimentalist, and other people in the local universe are all described quantum-mechanically by a density matrix. This density matrix

$$\rho(Q_{\rm loc}, Q'_{\rm loc}) = \int dQ_{\rm ext}\, \psi(Q_{\rm loc}, Q_{\rm ext})\, \psi^*(Q'_{\rm loc}, Q_{\rm ext})$$

results from integrating out the inaccessible degrees of freedom $Q_{\rm ext}$, e.g., those beyond the event horizon of the present accelerated cosmological expansion or the horizons of black holes. In addition, we delegate to $Q_{\rm ext}$ the environmental degrees of freedom that do not affect the experimental object and the macroscopic identity of the experimentalist and the other communicating observers in the local universe.

Given an underlying basic structure, we consider the set of all the emergent quantum configurations $\{\rho_r(Q_{\rm loc}, Q'_{\rm loc})\}$ that represent the beginning of the experiment and are macroscopically similar to each other in the observationally accessible patch of the universe. We call an individual mixed state r of this set a *realization* of the emergent physical world. After the experiment with several decoherent outcomes i, the density-matrix ρ_r of every realization r splits into a sum of density matrices for the Everett branches i:

$$\rho_r = \sum_i \rho_{ir}\,. \tag{206}$$

In this equation the omitted cross terms $\int dQ_{\rm ext}\, \psi_i\, \psi_j^*$ with $i \neq j$ vanish because the macroscopically distinct outcomes decohere. Then for the weights

$$w_r = \int dQ_{\rm loc}\ \rho_r(Q_{\rm loc}, Q_{\rm loc}) \equiv {\rm Tr}\, \rho_r\,, \tag{207}$$

$$w_{ir} = {\rm Tr}\, \rho_{ir} \tag{208}$$

we have

$$w_r = \sum_i w_{ir}\,. \tag{209}$$

We consider experimental conditions that at the beginning of the experiment unambiguously specify the relative wave function of the probed microscopic subsystem. Since the discussed realizations r are macroscopically similar in the local patch of the universe, they have the same relative wave function of the probed subsystem. Therefore, the ratios

$$\alpha_i = w_{ir}/w_r \tag{210}$$

are the same for every considered realization r. By eq. (209)

$$\sum_i \alpha_i = 1\,. \tag{211}$$

We introduce *cumulative distribution of the weights* for the locally-similar physical realizations ρ_r that describe the experiment beginning:

$$F(w) \equiv \left(\begin{array}{c} \text{number of realizations } \rho_r \\ \text{with } w_r \equiv {\rm Tr}\, \rho_r \geqslant w \end{array} \right). \tag{212}$$

Let $F_i(w)$ be the analogous cumulative distribution for the realizations of an outcome i after decoherence of the various outcomes of the experiment. For discrete non-degenerate outcomes $\{i\}$, $F_i(w)$ is related to the initial distribution (212) as

$$F_i(w) = F\left(w/\alpha_i\right). \tag{213}$$

(Indeed, a realization of the outcome i with weight w_{ir} is created by a realization of the initial state with weight w_{ir}/α_i.)

Similarly to branches ψ_i of a pure state ψ, a branch ρ_i of a mixed state ρ exists objectively if and only if $w_i \equiv {\rm Tr}\, \rho_i$ exceeds the threshold $w_{\rm min} = \delta\chi^2_{\rm min}$ of eq. (51), Sec. III. A local mixed configuration ρ splits into separate branches with smaller weights only through evolution of the degrees of freedom $Q_{\rm loc}$, contributing to the macroscopic identity of the local physical system. Branching of a pure wave function ψ due to evolution of $Q_{\rm ext}$, in particular, of the degrees of freedom beyond various event horizons, does not additionally split the local density matrix. This can be understood by viewing $Q_{\rm loc}$ and $Q_{\rm ext}$ as coordinates along independent dimensions of the basic distribution $\nu(q)$, depicted by Fig. 2 in Sec. IV C. In essence, we consider the same smooth functions of eq. (54) to fit *simultaneously* all the contributing to ρ_i patterns regardless of their position in the coordinates $Q_{\rm ext}$. It can be an objectively existing entity, with ${\rm Tr}\, \rho_i \gg \delta\chi^2_{\rm min}$, even if the wave functions that could describe any of the individual patterns have too low weight to exceed their objective existence threshold.

Also, as advertised in Sec. VII F, the trace over $Q_{\rm ext}$ removes the ambiguity of extending beyond the local Hubble volume the equal-time slice of spacetime that defines the system configuration or state. While choosing the equal-time hypersurface in the weakly perturbed Robertson–Walker metric is unambiguous locally (on sub-Hubble scales), on larger scales there are various "reasonable" yet non-equivalent choices, yielding qualitatively different conclusions for probabilities of specific physical configuration [57–60].

We return to the cumulative distributions $F(w)$ and $F_i(w)$ for the state weights before and after the experiment. The total number of the objectively existing state realizations with an outcome i is

$$N_i = F_i(w_{\rm min})\,, \tag{214}$$

as evident from the definition of cumulative distribution (212). The frequentist probability of the outcome i is therefore

$$P_i = \frac{N_i}{\sum_i N_i} = \frac{F_i(w_{\rm min})}{\sum_i F_i(w_{\rm min})}\,. \tag{215}$$

In particular, if an event or a chain of events becomes so unlikely that the respective N_i falls below unity then there are no physical outcomes of the type i. This makes P_i of eq. (215) the *objective physical probability* and distinguishes it from alternative formal assignments of probability, even if the latter are "rational" in the sense of Refs. [13–16].

For an experiment that splits quantum states into decoherent terms i of relative weights α_i (210), the probability (215) by eq. (213) becomes

$$P_i = \frac{F(w_{\min}/\alpha_i)}{\sum_i F(w_{\min}/\alpha_i)} \,. \tag{216}$$

For example, for a power-law cumulative distribution

$$F(w) = \frac{A}{w^p} \tag{217}$$

[with $p \geqslant 0$ so that $dF/dw \leqslant 0$ by $F(w)$ definition (212)] we have

$$P_i = \frac{\alpha_i^p}{\sum_i \alpha_i^p} \,. \tag{218}$$

Note that unless $p = 1$, the Born rule, requiring $P_i = \alpha_i$, does not apply. Likewise, the Born rule does not hold for any cumulative distribution $F(w)$ that is not described by a power law.

We now prove that the distribution $F(w)$ indeed has the power-law form with $p = 1$, hence the Born rule applies, under the following conditions. Consider an ensemble of states $\{r\}$ that are no longer required to be macroscopically similar. Let this ensemble be in equilibrium such that its cumulative distribution does not change by quantum splits in any subset of macroscopically similar states. This means that for any split

$$F(w) = \sum_i F_i(w) \,. \tag{219}$$

Then by eq. (213)

$$F(w) = \sum_i F\left(w/\alpha_i\right) \,. \tag{220}$$

For an experiment with two outcomes, of relative weights $\alpha_1 \equiv \alpha$ and $\alpha_2 = 1 - \alpha$, the above condition reads

$$F(w) = F\left(\frac{w}{\alpha}\right) + F\left(\frac{w}{1-\alpha}\right) \,. \tag{221}$$

Since the last equation should hold identically in α, we differentiate its both sides over α to obtain

$$\frac{1}{\alpha^2}\, f\left(\frac{w}{\alpha}\right) = \frac{1}{(1-\alpha)^2}\, f\left(\frac{w}{(1-\alpha)^2}\right) \tag{222}$$

where

$$f(w) \equiv -\frac{dF}{dw} \,. \tag{223}$$

The introduced function $f(w)$ is the weight distribution density for the considered ensemble of the emergent physical states. The identity (222) gives us $f = A/w^2$, where $A = \text{const}$. Integrating this result and fixing the integration constant by eq. (221), we determine the corresponding cumulative distribution of weights:

$$F(w) = \frac{A}{w} \,. \tag{224}$$

We noted after eq. (218) that cumulative distribution of the form (224) is special: It and only it yields the Born rule for the connection between the wave function and the probability for an observer to find oneself in the branch with a particular outcome.

There appears no obvious reason for the weight distribution to remain fixed over a time span during which cosmological parameters of the local universe change substantially. On the other hand, for quantum processes with shorter timescales the realizations of *typical* worlds may be in approximate equilibrium. Systematic investigation of the conditions that lead to the equilibrium (219) is beyond the scope of this paper.

B. Important real-world consequences

As time passes by, the quantum states that constitute our physical world incessantly split under numerous natural or human-driven processes. The natural processes include radioactive decays, stellar nuclear fusion, high-energy collisions of cosmic rays, etc. If a physical system incorporates regions where inflation is ending then its wave function undergoes an immense number of quantum splits during the creation of particles at reheating. Inflating regions also contribute to splitting by continuously foliating into branches with different classical values of the field modes that freeze after they exit the horizon.

The arguments of the previous subsection show that a non-zero minimal value of $w_i = \operatorname{Tr}\rho_i$ is essential for connecting the state norm with phenomenological probability. The positive lower limit on the squared norm w_i of a physically existing branch naturally follows from the first principles as shown in Sec. III. On the other hand, the positive cutoff for the norm has extremely disturbing real-world implications. Any physical state that represents our world is bound to split into progressively "thinner" branches each of which then eventually terminates.

Yet our world has already survived a tremendous amount of splits due to natural processes, including those listed above. We thus face two important questions:

1. What makes our world sustain natural splitting of the wave function?

2. May some relatively recent or future human-created processes cause splits that are more dangerous than the natural ones?

The preceding subsection IX A suggested an answer to the first question. The termination of some of the existing branches may be balanced by the appearance of new branches during the splits. Nonetheless, the sum of the weights $\sum_r w_r$ has to diminish during the evolution along a particular classical path, unless the ensemble of the states $\{r\}$ for this path replenishes from the appearance of new physical degrees of freedom from the Planck scale (Sec. VII E). No source of the replenishment is evident for our local universe in the current Hubble volume. Still, a quasi-equilibrium ensemble of states that evolve along a certain macroscopic path should not deplete exponentially fast for a *typical* evolution path, into which many states decay.

The quantum splits due to recent, by cosmological time-scales, human activity should be a minute fraction of all the splits. Yet some of these artificial splits, e.g. due to quantum computing, possibly never happened before. Reassuringly, the cause of the quantum split does not affect whether or not a given classical branch terminates as long as every followed decoherent branch has a weight of the order of that for its parent branch. Yet it is important whether or not our branch is sufficiently generic to continue existence for additional cosmologically significant duration of time.

It is easy to devise practically realizable selection of the branches that throws the observer to untypical, non-generic branches. For example, the Schrodinger cat can be arranged to survive only for an extremely unlikely but in principle possible quantum fluctuation. If we accept the Everett view of quantum evolution literally and believe that every outcome, with an arbitrarily small amplitude of its branch, exists then from the cat's point of view the animal could be expected to be safe. It is clearly not the case in the fundamental picture of physical evolution that is developed by this paper.

X. CONCLUSION

We considered the generic distribution of quantifiable properties of a large finite number of arbitrary static objects. We established that some low-resolution descriptions of this distribution can be identified with wave functions of emergent physical systems. These descriptions form a sequence of continuously evolving wave functions of elementary particle fields with local symmetries (e.g., gauge and diffeomorphism symmetries, resulting in gauge and gravitational interactions of the emergent fields).

The suitable collections of basic objects with many properties that can be quantified by real numbers are commonly encountered in mathematics and in the empirical world. Any such collection objectively contains emergent dynamical quantum fields with local symmetries, i.e., with evolution by physical laws similar to those of our observed world.

Independently from this, we showed that, regardless of the fundamental origin of quantum physics, the quantum superposition principle leads to the branches of evolution with any conceivable Hamiltonian for the physical fields. For the separate branches remaining decoherent, the alternative Hamiltonians should still possess pointer states [4, 7], stable to decoherence. Still, in general, there are many such different Hamiltonians, for example, those whose couplings adiabatically change at a rate that may also vary arbitrarily throughout time and space. We therefore face a question of why our daily experience and all the performed experiments agree on the same Hamiltonian of the Standard Model and general relativity with the constant values of its parameters.

We suggest a resolution by recalling that in a gauge- and diffeomorphism-invariant theory the arguments of the wave function are the whole orbits of the symmetry group [32]. A Hamiltonian that does not preserve the constancy of the wave function on these orbits cannot continuously transform the wave function while preserving the degrees of freedom of the physical system. In locally supersymmetric theories the requirement for a wave function to be constant on the symmetry orbits specifies fully the Hamiltonian, i.e., the future evolution of the system. Thus emergent systems with local supersymmetry possess unambiguous dynamical laws. These laws may differ among independently coexisting isolated emergent systems. But for any such system its Hamiltonian is uniquely encoded in the map of the generic basic structure to the orbits of the local supersymmetry. This map is what defines the emergent system. The Hamiltonian cannot change for the quantum states of that system during their evolution.

In the emergent physical system we manifestly see why quantum entanglement persists over arbitrarily large, even cosmological distances while the dynamics of the physical fields is strictly local. Entangled dynamical variables that refer to two widely separated physical objects are different coordinates of the generic basic distribution (in some of its representations). An entangled quantum state of these dynamical variables is simply represented by a wave function term that is localized in the both coordinates. Dynamical variables that we perceive as separated by tremendous distance are merely independent characteristics of the same feature of the underlying basic distribution. The physical distance is meaningful only in relation to the emergent local dynamics of the emergent quantum fields. In this perspective, quantum entanglement at any distance is trivial.

The reason for the locality of physical dynamics is less obvious. The dynamics is necessarily local for the emergent systems where some dynamical fields are parameters that quantify local transformations of other physical fields that emerge as described previously. This encompasses the physical modes of gauge connection fields—gauge bosons (Sec. V) and of the metric field—gravitons (Sec. VI).

The considered emergent quantum fields thus possess the standard gauge and gravitational interactions. The companion Ref. [33] demonstrates that even when the

emergent dynamical laws have no continuation beyond boundaries at which energy density or spacetime curvature approach the Planck scale, the physical evolution before the boundaries is unambiguous. This lets us answer several fundamental questions that involve physics at the Planck scale. Ref. [33] studies black holes and resolves the information [29] and firewall [31] paradoxes. Sec. VII E of the current paper investigates a related question of where the new microscopic degrees of freedom "appear from" when inflation stretches space perpetually and why all the short-scale modes are initially in the ground state of the system Hamiltonian.

We note fundamental connection among (i) stability of the physical vacuum to ultraviolet quantum loops, (ii) the initial state of inflationary modes, and (iii) black hole singularities. The new short-scale physical degrees of freedom that emerge during inflation should match to a unique description of the basic structure at scales were any excitation of field modes above their ground state collapses to a black hole. Moreover, after a black hole evaporates, it should end up at the physical vacuum, or the latter would be unstable under quantum fluctuations. Thus the same wave function of inhomogeneous modes should describe at these energies the vacuum (the ground state of the Hamiltonian of the emergent system), the initial state of the short-scale modes during inflation, and the final state of a black hole after its complete evaporation.

The same generic collection of objects with quantifiable properties contains isolated emergent quantum systems with fermionic fields and the supersymmetry. Similarly to the complex nature of the wave function (Sec. IV A), anticommutation of the fermionic field operators is not postulated but arises naturally. It may be technically convenient but by no means necessary to utilize the abstract Grassmann numbers in order to represent the emergent fermions. The fermionic states are fundamentally represented by the same generic collection of objects with properties that can be quantified by real or even natural numbers.

Various alternative formulations of quantum mechanics (for example, nine is identified in Ref. [61]) lead to macroscopic predictions most of which are indistinguishable for the existing experiments. Nevertheless, many of these formulations are *not mutually equivalent*. They suggest different objective, observer-independent organization of nature. Moreover, they lead to different predictions for some experiments that have not been performed but may be contemplated in principle.

The described emergent quantum evolution is necessarily Everettian. It branches into multiple coexistent non-communicating macroscopic worlds. However, for a finite underlying structure, the number of the objectively existing Everett branches, while possibly huge, is finite. As a result, the probability of a specific macroscopic outcome is well defined. Importantly, the branches whose norm diminishes below a fixed positive limit cease to exist as objective entities. Therefore, some outcomes that would be possible in the axiomatic quantum mechanics due to an unlikely but allowed quantum fluctuation do not materialize as physical reality.

The considered quantum states of evolving fields with gauge and gravitational interaction essentially are certain patterns in the arbitrary static arrangement of information. We may consider, for example, the information contained in a pile of sand grains or in bits recorded to a CD. Our results demonstrate that knowing the mathematical form of the physical laws [20] is insufficient for predicting some physical outcomes. The latter also depend on the material representation of this mathematics. Our physical world can be represented simultaneously by all the possible basic carriers of information.

The fundamental carriers of information are unlikely to be themselves the objects of our physical world, which in the suggested view emerges from the most general static structure. It is, therefore, difficult to speculate on the nature of these fundamental objects. On the other hand, we probably have all the tools for theoretical study of the physical systems that emerge in these structures generically. Along with experimentally accessible knowledge about our physical world, this may turn out to be sufficient for predicting the probabilities of the outcomes for every feasible physical process.

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Appendix A: Non-abelian gauge symmetry in curved spacetime

In this Appendix we consider the general locally Lorentz-invariant renormalizable gauge theory with scalar-field matter. We show explicitly that its wave function is unchanged by the gauge transformations of the fields. We allow for an arbitrary abelian or non-abelian gauge group and arbitrary $3+1$ dimensional spacetime metric $g_{\mu\nu}(x)$, treated in this Appendix as a classical external field.

The corresponding renormalizable, gauge-invariant and locally Lorentz-invariant Lagrangian density $\mathcal{L}$ involves spacetime derivatives of the gauge fields A^a_μ only through the covariant field-strength tensor

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu \,, \tag{A1}$$

where f^{abc} are the completely anti-symmetric structure constants of the gauge group. Hence the Lagrangian does not contain $\partial_t A_0^a$. Then the Euler-Lagrange equations for the fields A_0^a become the *primary constraint* equations

$$\hat{\pi}_a^0\,\psi = \frac{\partial \mathcal{L}}{\partial(\partial_t A_0^a)}\,\psi = 0\,, \tag{A2}$$

by which

$$\frac{\delta}{\delta A_0^a}\,\psi = i\hat{\pi}_a^0\,\psi = 0\,. \tag{A3}$$

Thus the wave function does not depend on A_0^a, in agreement with the construction of Sec. V.

The constancy of the wave function $\psi(\phi, A_i^a)$ on the gauge orbits follows from the secondary constraints, which can be obtained as follows. Gauge-invariant Lagrangian density $\mathcal{L}$ involves spacetime derivatives of the matter fields ϕ only through the covariant derivative

$$D_\mu\phi = (\partial_\mu - iA_\mu^a t^a)\phi\,. \tag{A4}$$

Here t^a are square matrices that multiply the matter fields column ϕ and generate its gauge transformation:

$$\delta\phi = i\varphi^a t^a \phi \tag{A5}$$

for infinitesimal transformation parameters $\varphi^a(x)$. The generators t^a form a representation of the Lie algebra of the gauge group:

$$[t^a, t^b] = i f^{abc} t^c\,. \tag{A6}$$

Denoting the fields ϕ, A_μ^a, and $g_{\mu\nu}$ by f, we thus have[14]

$$\mathcal{L} = \mathcal{L}(D_\mu\phi, F_{\mu\nu}^a, f)\,. \tag{A7}$$

The Lagrangian density $\mathcal{L}$ corresponds to the Hamiltonian density

$$\mathcal{H} = \dot{\phi}\cdot\pi + \dot{A}_i^a\pi_a^i - \mathcal{L}\,. \tag{A8}$$

From eq. (A4),

$$\dot{\phi}\cdot\pi = D_0\phi\cdot\pi + iA_0^a(t^a\phi)\cdot\pi\,, \tag{A9}$$

and from eq. (A1),

$$\dot{A}_i^a\pi_a^i = F_{0i}^a\pi_a^i + (\partial_i A_0^a - f^{abc}A_0^b A_i^c)\pi_a^i\,. \tag{A10}$$

For the Lagrangian density (A7), the canonical momenta fields π and π_a^i are:

$$\pi = \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = \frac{\partial\mathcal{L}(D_\mu\phi, F_{\mu\nu}^a, f)}{\partial(D_0\phi)}\,, \tag{A11}$$

$$\pi_a^i = \frac{\partial\mathcal{L}}{\partial\dot{A}_i^a} = 2\,\frac{\partial\mathcal{L}(D_\mu\phi, F_{\mu\nu}^a, f)}{\partial F_{0i}^a}\,. \tag{A12}$$

[14] In eq. (A7) and other equations below by displaying the dependence on $D_\mu\phi$ we also imply the dependence on $(D_\mu\phi)^\dagger$.

Eqs. (A11–A12) determine $D_0\phi$ and F_{0i} as functions of $(\pi, \pi_a^i, D_i\phi, F_{ij}^a, f)$. Then substitution of eqs. (A9–A10) to eq. (A8) yields

$$\begin{aligned}\mathcal{H} \;=\; & \mathcal{H}_N(\pi, \pi_a^i, D_i\phi, F_{ij}^a, f)\; + \\ & + A_0^a\left[i(t^a\phi)\cdot\pi - \partial_i\pi_a^i - f^{abc}A_i^b\pi_c^i\right],\end{aligned} \tag{A13}$$

where we dropped a total derivative $\partial_i(A_0^a\pi_a^i)$ and used the complete antisymmetry of f^{abc}.

The evolution $\partial_t\psi = -i\hat{H}\psi$ for the Hamiltonian density (A13) should not introduce the dependence of ψ on A_0^a, forbidden by the primary constraint (A3). This is the case only if the expression that multiplies A_0^a in eq. (A13) annihilates ψ:

$$\left[i(t^a\phi)\cdot\pi - \partial_i\pi_a^i - f^{abc}A_i^b\pi_c^i\right]\psi = 0\,. \tag{A14}$$

It is the *secondary constraint.*

Given the gauge variation of ϕ from eq. (A5) and respectively

$$\delta A_i^a = \partial_i\varphi^a + f^{abc}A_i^b\varphi^c\,, \tag{A15}$$

we have

$$\begin{aligned}\delta_{\text{gauge}}\psi \;&=\; \int d^3x\left(\delta\phi\cdot\frac{\delta\psi}{\delta\phi(\boldsymbol{x})} + \delta A_i^a\frac{\delta\psi}{\delta A_i^a(\boldsymbol{x})}\right) = \\ &= i\int d^3x\,\varphi^a\left[i(t^a\phi)\cdot\pi - \partial_i\pi_a^i - f^{abc}A_i^b\pi_c^i\right]\psi\,.\end{aligned}$$

The expression of the last line vanishes due to the secondary constraint (A14). Thus for the general gauge-invariant and Lorentz-invariant local action, the wave function is constant along the gauge-group orbits:

$$\delta_{\text{gauge}}\psi = 0\,. \tag{A16}$$

Appendix B: Generally covariant Hamiltonian of matter

This Appendix presents the Hamiltonian density that by the Legendre transformation (A8) corresponds to the general gauge- and diffeomorphism-invariant bosonic action (161) with renormalizable non-gravitational part. The Appendix also explicitly verifies that the respective operator $\int d^3x\,N^i\mathcal{H}_i$ generates Lie translations of the physical fields in space.

For the gauge part of the action $S^A = \int d^4x\,\mathcal{L}^A$ with

$$\mathcal{L}^A = \int d^4x\,\frac{\sqrt{-g}}{4e^2}\,F_{\mu\nu}^a F^{a\,\mu\nu} \tag{B1}$$

we find after calculation that

$$\begin{aligned}\mathcal{H}^A \;&=\; \dot{A}_i^a\pi^{ai} - \mathcal{L}^A \;= \\ &=\; N^\alpha\mathcal{H}_\alpha^A + (\partial_i A_0^a + f^{abc}A_i^b A_0^c)\pi^{ai}\end{aligned} \tag{B2}$$

where explicitly

$$\mathcal{H}_N^A = \frac{e^2}{2\sqrt{\gamma}}\,\pi^{ai}\gamma_{ij}\pi^{aj} + \frac{\sqrt{\gamma}}{4e^2}\,\gamma^{ik}\gamma^{jl}F^a_{ij}F^a_{kl}\,, \quad \text{(B3)}$$

$$\mathcal{H}_i^A = F^a_{ij}\pi^{aj}\,. \quad \text{(B4)}$$

For scalar fields the renormalizable Lagrangian density with the required symmetries is

$$\mathcal{L}^\phi = -\sqrt{-g}\,[g^{\mu\nu}(D_\mu\phi)\cdot D_\nu\phi + V(\phi)]\,, \quad \text{(B5)}$$

where ϕ is a column of real scalar fields ϕ^α (here we present any complex field as a pair of real ones), D_μ is the gauge-covariant derivative (A4), and a potential $V(\phi)$ is invariant under the gauge transformations of ϕ. The corresponding Hamiltonian density equals

$$\begin{aligned}\mathcal{H}^\phi &= \dot\phi\cdot\pi - \mathcal{L}^\phi = \\ &= N^\alpha\mathcal{H}^\phi_\alpha + A^a_0 j^{a0}\end{aligned} \quad \text{(B6)}$$

with

$$\mathcal{H}_N^\phi = \frac{\pi\cdot\pi}{2\sqrt{\gamma}} + \sqrt{\gamma}\left[\frac{1}{2}\gamma^{ij}(D_i\phi)\cdot D_j\phi + V(\phi)\right], \quad \text{(B7)}$$

$$\mathcal{H}_i^\phi = (D_i\phi)\cdot\pi \quad \text{(B8)}$$

and, by eqs. (122, A5),

$$j^{a0} = i(t^a\phi)\cdot\pi\,. \quad \text{(B9)}$$

Due to the secondary gauge constraint (A14) and the asymmetry of f^{abc}, the sum of the terms that involve A_0 in eqs. (B2, B6) annihilates ψ. The remaining terms and the gravitational part (163) give for the total Hamiltonian of the action (161)

$$H\psi = \int d^3x\, N^\alpha\mathcal{H}_\alpha\psi \quad \text{(B10)}$$

where

$$\mathcal{H}_\alpha = \mathcal{H}^g_\alpha + \mathcal{H}^A_\alpha + \mathcal{H}^\phi_\alpha\,, \quad \text{(B11)}$$

with $\mathcal{H}^g_\alpha \equiv (\mathcal{H}^N_g, \gamma_{ij}\mathcal{H}^j_g)$.

Let us verify from the explicit results above that $\int d^3x\, N^i\mathcal{H}_i$ generates the Lie translations of all the elementary physical fields in space [eq. (151) for $\mu \neq 0$]. Indeed, by eq. (166) and eqs. (159, 164),

$$\begin{aligned}\int d^3x\, N^i\mathcal{H}^g_i &= \int d^3x\, N_{(i|j)}\,\pi^{ij} = \\ &= -i\int d^3x\, L_{\boldsymbol{N}}\gamma_{ij}\,\frac{\delta}{\delta\gamma_{ij}}\,.\end{aligned} \quad \text{(B12)}$$

By eqs. (B4) and (131),

$$\begin{aligned}\int d^3x\, N^i\mathcal{H}^A_i &= \int d^3x\, N^i F^a_{ij}\,\pi^{aj} = \\ &= -i\int d^3x\,[-iL_{\boldsymbol{N}}A^a_j\,\frac{\delta}{\delta A^a_j} + N^i A^a_i j^{a0} + C]\,.\end{aligned} \quad \text{(B13)}$$

where $C\psi = 0$, and where for the second equality we integrated the term $\int d^3x\, N^i(-\partial_j A^a_i)\pi^{aj}$ by parts and applied the secondary gauge constraint (A14). Finally, for the scalar fields we find from eqs. (B8) and (19) that

$$\begin{aligned}\int d^3x\, N^i\mathcal{H}^\phi_i &= \int d^3x\, N^i(\phi_{,i}\cdot\pi - A^a_i j^{a0}) = \\ &= \int d^3x\,[-i(L_{\boldsymbol{N}}\phi)\cdot\frac{\delta}{\delta\phi} - N^i A^a_i j^{a0}]\,.\end{aligned} \quad \text{(B14)}$$

Adding up these equations, we obtain

$$\int d^3x\, N^i\mathcal{H}_i = -i\int d^3x \sum_{f=\gamma_{ij},A^a_i,\phi^\alpha} L_{\boldsymbol{N}}f\,\frac{\delta}{\delta f(\boldsymbol{x})}\,. \quad \text{(B15)}$$

This confirms that the operator $\int d^3x\, N^i\mathcal{H}_i$ generates Lie translations of the fields in space:

$$e^{-i\int d^3x\, N^i\mathcal{H}_i}\psi(f) = \psi(f - L_{\boldsymbol{N}}f)\,. \quad \text{(B16)}$$

Appendix C: Quasiclassical limit of quantum gravity in Hamiltonian description

Lapchinsky and Rubakov [41] considered solutions of the Wheeler-DeWitt equation with matter (174) in which the gravitational degrees of freedom are confined to the classical configurations of constructive interference [42] while the matter fields are treated fully quantum-mechanically. They showed that in the leading order of quasiclassical expansion the prefactor $\tilde\psi(\tilde f, \bar f_{\rm cl}(t))$ of the quasiclassical ansatz (176) for such solutions satisfies

$$i\partial_t\tilde\psi = \tilde H\tilde\psi\,. \quad \text{(C1)}$$

This formula has the appearance of the Schrodinger equation. Yet several obstacles, discussed further, should be overcome in order to reduce it to the actual Schrdinger equation of quantum field theory.

Kim [43] later demonstrated that the conditions for quasiclassicality necessarily fail when in an expanding universe only gravity is treated quasiclassically. In this Appendix we extend the arguments of Lapchinsky and Rubakov to our case, where the quasiclassical variables $\bar f$ are the long-wavelength components of the metric and matter fields together.

We start from the exact Hamiltonian constraint for the full wave function (176):

$$(\bar H + \tilde H)\left[A(\bar f)\,e^{iS(\bar f)}\,\tilde\psi(\bar f, \tilde f)\right] = 0\,. \quad \text{(C2)}$$

From application of $\bar\pi = -i\delta/\delta\bar f$, entering $\bar H(\bar f, \bar\pi)$, we retain only[15] the first derivatives of S and $\tilde\psi$ over $\bar f$. We

[15] The derivatives of $A(\bar f)$ are balanced at the higher orders of the quasiclassical expansion for the background $\bar\psi(\bar f) \equiv A\,e^{iS}$, eq. (178). Hence they do not affect the short-scale wave function $\tilde\psi$ in the considered leading order.

then have

$$\bar{H}(\bar{f}, \frac{\delta S}{\delta \bar{f}})\,\tilde{\psi} - i\int d^3x\, \frac{\delta \bar{H}}{\delta \bar{\pi}_\alpha(\boldsymbol{x})}\, \frac{\delta \tilde{\psi}}{\delta \bar{f}^\alpha(\boldsymbol{x})} + \tilde{H}\tilde{\psi} = 0\,. \quad \text{(C3)}$$

The first term in eq. (C3) vanishes by eq. (180), which defined $S(\bar{f})$. If we confine $\bar{f}$ to the configurations $\bar{f}_{\rm cl}$ then by eq. (186) we can replace $\delta\bar{H}/\delta\bar{\pi}_\alpha(\boldsymbol{x})$ in the second term by $\dot{\bar{f}}^\alpha_{\rm cl}(\boldsymbol{x})$. Then eq. (C3) becomes

$$-\,i\int d^3x\, \dot{\bar{f}}^\alpha_{\rm cl}(\boldsymbol{x})\, \frac{\delta\tilde{\psi}(\tilde{f}, \bar{f}_{\rm cl})}{\delta \bar{f}^\alpha_{\rm cl}(\boldsymbol{x})} + \tilde{H}\tilde{\psi} = 0\,. \quad \text{(C4)}$$

This gives

$$i\partial_t\tilde{\psi}(\tilde{f}, t) = \tilde{H}\tilde{\psi}(\tilde{f}, t) \quad \text{(C5)}$$

where

$$\tilde{\psi}(\tilde{f}, t) \equiv \tilde{\psi}(\tilde{f}, \bar{f}_{\rm cl}(t))\,. \quad \text{(C6)}$$

Eq. (C5) reminds us the Schrodinger equation for a time-dependent wave function $\tilde{\psi}(\tilde{f}, t)$ of the "microscopic" degrees of freedom $\tilde{f}$. However, its "Hamiltonian" $\tilde{H}$ involves not only the momenta of matter fields $\tilde{\phi}$ and $\tilde{A}_i$ but also of the metric field $\tilde{\gamma}_{ij}$. Moreover, the metric scale factor $a = \bar{a} + \tilde{a}$ enters the kinetic energy term $\delta^2/\delta\tilde{a}^2$ of this Hamiltonian with the "wrong" sign, evident from eq. (174). This does not yet imply vacuum instability because $\tilde{a}(\boldsymbol{x})$, along with three other independent parameters of $\tilde{\gamma}_{ij}$, can be gauged away by a change of four spacetime coordinates x^μ. Nonetheless, this obstacle needs to be resolved with some consistent procedure.

There is another difficulty in using eq. (C5) for actual calculations. We defined $\bar{f}_{\rm cl}(t)$ as the trajectory of constructive interference for the background Hamiltonian $\bar{H}$. However, the added Hamiltonian $\tilde{H}$ for the short-wavelengths modes, typically of the form $\tilde{H}(\tilde{f}, \tilde{\pi}, \bar{f})$, displaces the constructive interference trajectory away from $\bar{f}_{\rm cl}(t)$. Indeed, in the Hamilton equation for $\dot{\bar{\pi}}$ the classical force $-\delta\bar{H}/\delta\bar{f}$ should then be corrected by the additional contribution $-\delta\tilde{H}/\delta\bar{f}$.

It is possible to provide a formal stable solution to the equation $(\bar{H} + \tilde{H})\,\psi(\bar{f}, \tilde{f}) = 0$ by expanding $\psi(\bar{f}, \tilde{f})$ over the eigenstates of $\tilde{H}$ [62] or over another convenient complete set of states [43] as

$$\psi(\bar{f}, \tilde{f}) = \sum_n c_n(\bar{f})\,\psi_n(\bar{f}, \tilde{f})\,. \quad \text{(C7)}$$

The resulting formalisms [43, 62], however, become more involved than just a single Schrodinger equation of the form (C1). To the contrast, the Lagrangian approach, pursued in Sec. VII C of the main text, yields compact evolution equations with simple physical interpretation.

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